Accountability in Governing Hierarchies*

Christopher M. Li† Greg Sasso‡ Ian R. Turner§

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Abstract

Formal theories of accountability and bureaucratic politics typically consider voter-politician interactions in isolation from politician-bureaucrat interactions. In this paper, we study a model of electoral accountability and policymaking with a hierarchy consisting of a voter, a politician, and a bureaucrat. The politician and bureaucrat both produce government output valued by the voter. The voter can then choose to reelect the politician, while the politician can expropriate some of the bureaucrat's output for his own ends. We show that when times are conducive to high quality governance – budgets are large and players are farsighted – incorporating the bureaucratic layer of the hierarchy makes for weaker accountability standards. However, when times are tough and budgets are small or players are myopic it is possible that voters may benefit from increasing their demands on elected officials. These accountability standards change even when reelection does not depend at all on the bureaucrat's output directly.

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Comments welcome.

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†Assistant Professor, Department of Political Science, Florida State University, Email: cli5@fsu.edu.

‡Postdoctoral Associate, Bocconi University, Email: gregory.sasso@unibocconi.it.

§Assistant Professor, Department of Political Science, Yale University, Email: ian.turner@yale.edu.

1 Introduction

A central tenet of effective democratic governance is citizens' ability to hold government officials accountable. In the United States, the central institution for doing so is popular elections. When voters elect their representatives they do so knowing that if elected officials fail to act in accordance with public wishes they can vote them out of office in the next election. Ideally, this provides political incentives for representatives to act in accordance with citizen interests.[1] However, since elected officials possess expertise and information voters do not and have a large influence on government policy, the so-called *political agency problems* arise when there are conflicts of interest between citizens and their representatives.[2] The worry is that elected politicians will use their superior expertise to pursue government action that runs contrary to public interest.

To complicate matters further, in most modern democracies government output is jointly produced by elected officials and unelected bureaucrats. This introduces another layer of accountability between the politician and bureaucrat, who also may have conflicting policy preferences. Of course, elected representatives usually have a measure of control over the bureaucrats through retention decisions, budget allocations, or project assignments. However, just as the politician often has an expertise advantage over citizens the bureaucracy often houses more expertise, or capacity to develop expertise, relative to politicians, which introduces the possibility for subversion by unelected bureaucrats (e.g., Gailmard 2002). Thus, there is another link in the chain of accountability susceptible to political agency problems, even if elected officials act faithfully on behalf of their constituents.

Much of the existing theoretical work on democratic accountability focuses one level of accountability: citizen-politician or politician-bureaucrat.[3] In this paper we analyze both layers of accountability simultaneously to explore how explicitly taking into account the politician-bureaucrat

[1] See Barro (1973), Ferejohn (1986), Fiorina (1981), Key (1966), and Manin (1997), among many others.

[2] Besley (2006) provides a nice review of political agency.

[3] See Ashworth (2012) for a comprehensive review of research focused on electoral accountability and Gailmard and Patty (2012) for a review on formal models of bureaucracy, which speaks to the politician-bureaucrat accountability relationship.

relationship alters the dynamics of electoral accountability (i.e., the citizen-politician relationship). Specifically, to better understand the dynamics of this 'governing hierarchy' we analyze an infinite horizon political agency model with a representative voter, an elected politician, and an unelected bureaucrat. The voter and bureaucrat are long-lived, approximating something akin to civil service protections in the case of the bureaucrat. The politician remains in office only as long as the voter continues to reelect him, introducing canonical concerns of electoral accountability.

In every period the bureaucrat invests effort toward producing government output (e.g., public service provision, regulatory enforcement, policy development). Bureaucratic output is more valuable when bureaucrats work hard in its production.[4] The politician then directs bureaucratic output towards two projects that are both valued equally by the voter. However, there is a conflict of interest between the politician and bureaucrat in terms of how the bureaucrat's capacity should be directed. To highlight its effect, we consider the most stark scenario where the bureaucrat cares exclusively about one of the projects while the politician cares about the other. For example, the bureaucrat might be committed to his agency's mission and care about devoting his expertise to furthering that mission while the politician might have more party-specific, targeted programs to prioritize for political reasons that cut against the agency's mission.

The politician also unilaterally decides how to allocate a "budget". He can allocate some or all of the budget to public goods provision that benefits citizens while the remainder is diverted toward private goods that solely benefit the politician. The voter reelects the incumbent based on his performance in providing public goods the voter. If the politician produces public goods sufficient to satisfy voter demands he is reelected and if he does not then he is thrown out of office. If the politician fails to be reelected, an identical politician takes his place in the following period. The game consists of an infinite number of periods.

We show that the need to sustain cooperation between the politician and the bureaucrat feeds back into the optimal level of electoral accountability. That is, by explicitly incorporating the full

[4]Conceptually, this is similar to models of policy development, or valence, and bureaucratic capacity such as Hirsch and Shotts (2012, 2015, 2018); Sasso (2019); Ting (2009); Turner (2017, 2019*a*,*b*). While our theory does not analyze the trade-offs between spatial policymaking and quality or valence as in much of this literature, the common value nature of bureaucratic output in our model follows in this spirit.

governing hierarchy we show that, relative to a setting with only voter-politician interactions, sometimes the additional consideration of politician-bureaucrat interactions leads to weakened accountability standards while in other environments voters are optimally more demanding of their elected representatives.

In a benchmark setting with no bureaucrat the voter is concerned only with the politician's performance, which depends on two competing forces, similar to the logic in Ferejohn (1986). On one hand, if she demands high levels of public goods, and the budget is sufficient for the politician to provide them, then the voter benefits because lots of public goods will be produced. On the other hand, demanding too much is risky. If the politician does not have the resources sufficient to satisfy voter demands then he will instead divert the entire budget away from public goods and accept that he will be voted out of office. While less stringent voter demands increase the likelihood that the politician will have the resources to provide public goods sufficient for reelection a lower level of goods will be provided. During times of relative political-economic prosperity the expected budget is high and the voter can focus on the first force. When times are tough and the expected budget is low the voter must worry about the second force. Ultimately, when there is no additional level in the governing hierarchy the voter balances these two forces when setting her accountability standards.

Adding a bureaucratic layer to the hierarchy introduces additional considerations. Now the voter is also concerned with the bureaucrat's output and, thus, the nature of the politician-bureaucrat relationship is salient for the optimal electoral accountability standards. Even though the voter has no direct control over the bureaucrat, her ability to impact the politician's tenure in office indirectly structures the incentives for cooperation between the politician and bureaucrat. Consider the politician and bureaucrat in isolation. The bureaucrat invests effort toward government output that the politician would like to expropriate for his own purposes. The politician cannot formally commit to *not* expropriating all of the bureaucrat's output. Indeed, in a world in which the politician and bureaucrat interact only once, that is exactly what the politician would do. Were that the case, the bureaucrat would never invest in expertise lest he pay the cost of doing so with no concomitant benefit. However, if it was certain that the politician needs to interact with the bureaucrat repeat-

edly then the long-term nature of the relationship would allow for cooperation even though neither party can credibly commit ex ante to doing so. That is, there is a *relational contract* between the politician and the bureaucrat that captures the implicit relations based on trust between the players.[5] Because the politician wants the bureaucrat to invest again in the future, he is willing to forego total expropriation in order to preserve a productive relationship in the future. In turn, the bureaucrat can safely invest more effort into government output, which benefits the voter.

The voter's accountability standards directly impact the politician's expected tenure in office, which in turn affects both the stability and the productivity of the politician-bureaucrat relationship as described above. Stringent standards weaken expectations that the politician will hold office for long, which increases the likelihood that the relational contract between the politician and bureaucrat will break down. Lax standards strengthen the politician's electoral expectations, which allows for a more productive long-term politician-bureaucrat relationship. Compared to the benchmark in which the voter only needed to concern herself with the politician's public goods provision, now the dynamics of the politician-bureaucrat relational contract tend to incentivize the voter to *weaken* her accountability standards and require less of the politician. This is because doing so strengthens the relational contract by improving expectations that the politician will remain in office, which, in turn, increases bureaucratic output. However, including the bureaucrat also increases the politician's returns to holding office because it introduces a secondary source for private rents. Since the politician can expropriate some of the bureaucrat's output for his private gain, in addition to the portion of the budget he may be able to successfully divert, remaining in office is more lucrative than the benchmark setting. This tends to push the voter to *strengthen* her accountability standards and demand more from the politician since now he is more willing to do so to ensure reelection.

Which effect of the politician-bureaucrat relationship dominates depends on the nature of the political-economic environment. Specifically, we show that when times are conducive to high quality governance – budgets are large and players are farsighted – incorporating the bureaucratic layer of the hierarchy makes weaker accountability standards more normatively desirable. However,

[5] See, for example, Levin (2003).

when times are tough and budgets are small or players are myopic it is possible that voters may benefit from increasing their demands on elected officials, relative to the benchmark setting. The value of maintaining a long-term, productive relationship with the bureaucrat is lucrative for the politician and he is more willing to satisfy the voter's demands to continue that relationship. Overall, these dynamics suggest that in order to adequately characterize optimal electoral accountability it is imperative to explicitly incorporate the varied relationships within the larger governing hierarchy.

Related literature. There's a large literature on democratic accountability, but most of that literature focuses on only one relationship: voter-politician or politician-bureaucrat. The current literature on electoral accountability has mostly focused on issues of selection at the expense of issues of moral hazard. In these models, voters want a good politician where "good" can represent many things such as competence, ideology, or honesty. For example, in models of pandering (Maskin and Tirole (2004); Morelli and Van Weelden (2013); Fox and Van Weelden (2012); Kartik and Weelden (2019)),[6] voters are unsure about which policy is correct. Politicians choose policies that maximize the voter's belief that the politicians are the good-type (in terms of competence, honesty, or ideology), even if that is not the best policy for voter welfare. In multitask models (Ashworth and Mesquita (2006); Daley and Snowberg (2009)), politicians choose between multiple actions with some actions better for voters than others. However, inefficient actions may be more informative than efficient actions in signaling politician type. Effort is therefore diverted from efficient actions to inefficient actions due to reelection concerns.[7]

The small but burgeoning literature on hierarchical accountability has also generally built upon the adverse selection accountability models. In particular, the current literature has used the pandering setup to great effect. Vlaicu and Whalley (2016) shows how the voter can use an intermediary to mitigate a policymaker's pandering incentives. Ujhelyi (2014) analyzes tensions between improving selection of politicians and selection of bureaucrats. In Fox and Jordan (2011), a misaligned politician delegates to a bureaucrat in order to avoid revealing her type to the voter, which

[6]For comprehensive treatments of pandering see Canes-Wrone, Herron and Shotts (2001) and, for accountability pathologies more generally, see Gersen and Stephenson (2014).

[7]See Ashworth (2012) and Duggan and Martinelli (2017) for more extensive surveys of the literature.

may enhance her reelection prospects.

Similarly, Yazaki (2018) shows that aligned bureaucrats can credibly signal the politician's type to the voter, improving electoral selection. Hirsch and Kastellec (2019) study settings in which third party actors, such as disgruntled bureaucrats, might 'sabotage' incumbent policymaking efforts in order to harm voters' ability to learn about incumbent quality. Finally, Li (2019) shows how voters can use their control over the politician to hold a political appointee accountable when they have good information about the appointee's type.

We complement these analyses by studying the hierarchical moral hazard problem rather than focusing on the adverse selection problem.[8] To that end, our model follows in the tradition of Barro (1973) and Ferejohn (1986), and focuses on the voter's control over the politician. Our contribution is to show how this control changes in the presence of a bureaucrat. As all possible politicians are the same, electing a different politician is purely an effort at control, not selection. The presence of a bureaucrat can increase or decrease how much the voter can demand from the politician. Optimal electoral accountability depends on the nature of the governing hierarchy.

On the politician-bureaucrat side, much of the existing literature has studied when and how delegation to bureaucrats is structured (McCubbins, Noll and Weingast (1987, 1989)), including how much discretion is optimal (Epstein and O'Halloran (1994, 1999); Huber and Shipan (2002)), expertise acquisition (Gailmard and Patty (2007), and how to control bureaucrats once authority has been transferred (Patty and Turner (2019)).[9] While we do not model these politician-bureaucrat dynamics in as rich a setting as those that focus on that relationship specifically, we do capture politician-bureaucrat conflicts-of-interest and the need for cooperation. For instance, one can conceptualize the politician's decision to allow the bureaucrat to direct his effort toward causes he intrinsically values – as opposed to the politician fully directing how bureaucratic output is utilized – as a

[8]There are a few hierarchical papers that also do not focus on adverse selection models. For example, Forand (2018) uses a hierarchical setup to look at the effects of civil service reform. While he also studies a moral hazard problem, his paper focuses primarily on the politician-bureaucrat relationship and competition between parties as opposed to the voter's behavior. Ellis and Groll (2019) applies the three-tiered principal-agent setup to lobbying with and without intermediaries. Intermediaries help with information transmission to politicians when the interest group's policy area is low salience.

[9]See Miller (2005), Gailmard and Patty (2012), and Moe (2012) for extensive surveys of the literature.

simple way of capturing bureaucratic discretion in a cooperative politician-bureaucrat relationship. If the politician expropriates all of the bureaucrat's output then there is no discretion and, in line with previous work, there is then no ex ante incentive for the bureaucrat to invest effort. Our main focus, however, is on how the *presence* of such a relationship might (or, might not) alter the nature of electoral accountability. That is, our contribution lies in the dynamic linkages we characterize between the politician-bureaucrat and the voter-politician relationships.

Finally, as described above, the politician-bureaucrat relationship in our model is built upon the theory of implicit or relational contracts. MacLeod and Malcomson (1989) provides the definitive treatment of relational contracts with symmetric information. Our model is conceptually similar to Baker, Gibbons and Murphy (2002) and Levin (2003), as the agent exerts unobservable effort without an explicit enforceable contract. As in Halac (2012), the politician has private information about his outside option (in this case, taking all of the budget for himself). We build on these models by showing how the uncertainty of the electoral process impacts the stability of the relational contract. If the voter is too demanding, and the politician's hold on office becomes tenuous the value of the contract is reduced for both parties.

2 A hierarchical model of accountability

We study an infinite horizon political agency model where periods are indexed $t = 1, 2, \ldots$. There are three players: a long-lived voter, a long-lived bureaucrat, and an elected politician who remains in office until voted out. At the beginning of each period a budget b_t is drawn i.i.d. from a continuous, log-concave distribution F with density $f(\cdot)$. The budget is only observed by the politician. The politician chooses how to allocate the budget toward public goods valued by the voter, denoted by $g_t \geq 0$, and 'consumes' the rest. While we are agnostic about what the politician's consumption of the budget represents specifically, it can generally be thought of as the politician directing the money toward projects that he values and the voter does not, perhaps privately targeted goods. In some settings it may also represent straightforward corruption. We will refer to the portion of the budget directed away from voter interests by the politician generally as 'private goods.' Formally,

$g_t \leq b_t$ captures the level of public goods produced by the politician and $x_t \coloneqq b_t - g_t$ denotes the amount of the budget directed toward private goods by the politician.

The bureaucrat also produces government output valued by voters. This is captured by $e_t \geq 0$ with a marginal cost of effort $c(e_t) \coloneqq \frac{e_t^2}{2\lambda}$. The politician can reward (or discipline) the bureaucrat based on output. Specifically, the politician chooses $\alpha_t \in [0,1]$ of output e_t to give to the bureaucrat (i.e., the bureaucrat receives $\alpha_t e_t$). The politician keeps $(1-\alpha_t)e_t$ for himself. Regardless of α_t, the voter receives all of e_t so α only directly affects the payoffs of the politician and bureaucrat. This 'sharing rule' can be interpreted as the bureaucratic output being divided between two settings or goals. Share α goes towards something the bureaucrat intrinsically cares about, perhaps policies and programs that are aligned with and further the agency's mission, while $1-\alpha$ goes towards something the politician cares about, perhaps for the purposes of party-building or the like. More generally, this interaction captures broad scenarios in which there are conflicts of interest between the elected official and the bureaucrat. The politician has the final say over how agency resources are deployed once they are developed, but the bureaucrat derives leverage from his role as the sole developer of said expertise. The voter is indifferent between the two possibilities and gets e_t regardless of how the output is divided. Conceptually, α represents a tool for the politician to reward, or punish, the bureaucrat based on administrative effort toward government policy production.

The politician's public goods production g_t and the bureaucrat's output e_t jointly produce government outcomes that affect the voter. Accordingly, following the politician's and bureaucrat's output choices the voter chooses to either reelect the incumbent politician or remove him from office. We analyze equilibria in which the voter employs a retrospective cut-off reelection strategy in public goods provision, g_t. Formally, this means that the voter reelects the incumbent if and only if $g_t \geq \gamma_t$ where γ_t denotes the minimum level of public goods the voter must receive to reelect the incumbent. The voter does not observe the bureaucrat's output e_t and, therefore, does not condition her reelection strategy on e_t directly. If the incumbent politician is voted out then he is replaced by a challenger and the game continues. To summarize, the timing of each period t proceeds as follows.

1. The budget b_t is drawn according to F and observed by the politician.

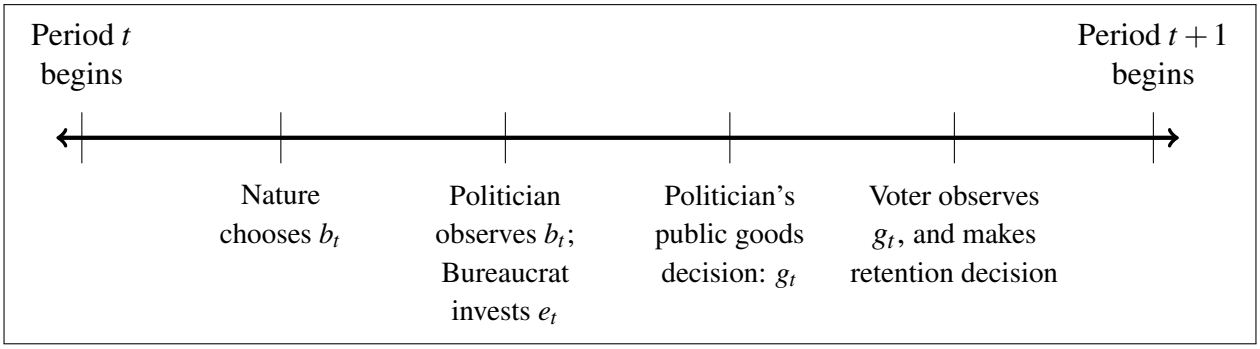

Figure 1: Game Timing

2. The bureaucrat produces output e_t at cost $c(e_t) \coloneqq \frac{e_t^2}{2\lambda}$.

3. The politician observes e_t and chooses how much of the budget to allocate to public goods, g_t, and how much to privately consume, $x_t \coloneqq b_t - g_t$, and how much to reward the bureaucrat, $\alpha_t e_t$, and how much to expropriate for himself, $(1-\alpha_t)e_t$.

4. The voter observes g_t and chooses to reelect the incumbent or not.

Figure 1 displays the sequence of play in each period graphically.

Payoffs. The voter maximizes the discounted sum of public goods and bureaucratic output:

$$U_V(g_t, e_t) = \mathbb{E}\sum_{t=1}^{\infty} \delta^{t-1}(g_t + e_t).$$

Each period t the voter wants to maximize the joint government output from the politician and the bureaucrat $(g_t + e_t)$, but her only tool for doing so is her reelection threshold γ_t.[10]

The politician maximizes the discounted sum of private rents, both from his own budget and from the bureaucrat's output while also striving to be reelected:

$$U_P(x_t, \alpha_t) = \mathbb{E}\sum_{t=1}^{\infty} \delta^{t-1}\mathbb{I}_t \times (x_t + (1-\alpha_t)e_t).$$

[10]In the Appendix we include additional analysis in which the voter differentially values public goods versus bureaucratic output. The main qualitative insights we focus on in the main text remain unchanged. Details can be found in Section A.3.

$\mathbb{I}_t$ is the indicator function and takes on value 1 if the politician is in office and 0 otherwise.[11] While the politician would like to direct the budget toward targeted private benefits at the expense of public goods and expropriate the bureaucrat's effort for other purposes he also wants to remain in office, which may restrict his ability to redirect government output for private gain.

Finally, the bureaucrat seeks to maximize the discounted sum of rewards from his output minus the costs of effort:

$$U_B(e_t, \lambda) = \mathbb{E} \sum_{t=1}^{\infty} \delta^{t-1} \left(\alpha_t e_t - \frac{e_t^2}{2\lambda} \right).$$

While the bureaucrat cannot directly affect his share of output, his optimal output decision, e^*, may impact the politician's decision calculus with respect to α.

Equilibrium. The voter's strategy is a collection of mappings, one for each period, from public histories (i.e., past actions by all players) to a decision whether to reelect the politician. The politician's strategy is a collection of mappings from public histories (i.e., past realizations of the budget and actions by all players) to the interval $[0, b_t]$ where b_t is the current realization of the budget. The bureaucrat's strategy is a collection of mappings from public histories (i.e., past actions by all players) to the positive reals.

Subgame perfect equilibrium is the natural solution concept in this setting. As in Ferejohn (1986), we will focus on a particular subclass of the SPE: those in stationary strategies, which imply the following in terms of player strategies. The equilibrium strategy of the voter is such that the reelection threshold in terms of public goods is constant in every period. The equilibrium strategy of the politician is such that the sharing rule (i.e., the proportion of bureaucratic output the politician appropriates conditional on her meeting the reelection threshold) is constant in every period. Finally, the bureaucrat's equilibrium strategy is such that he exerts constant effort in every period. Given stationarity, we can drop time subscripts when describing equilibrium actions and reserve them for other uses. Thus, the notation in the style of γ^* denotes equilibrium reelection thresholds. Similarly, α^* and e^* denote an equilibrium sharing rule by the politician and equilibrium effort by the bureaucrat, respectively.

[11] We could also include a fixed 'ego rent' from remaining in office without qualitatively altering the results.

One justification for focusing on equilibrium in stationary strategies is that it places the least amount of "mental burden" on the players; instead of devising strategies that depend on past histories in a complicated way, the players simply follow the same action repeatedly. Practically, the advantage of stationary equilibrium is that together with some welfare criterion, it allows for comparative statics.

Comments on modeling assumptions. It is worth taking a moment to consider a few important assumptions from the model set-up. The term "budget" need not be taken to mean government revenue literally. One can imagine a variety of resources, monetary or otherwise, that politicians control and that are imperfectly observed by the public. The politicians can then either utilize such resources for the public good or for their private gains. For example, politicians can direct their time and attention to public policy or to campaigning for the next election. Relatedly, the interpretation of "public goods" can be taken to include any measure of the politician's performance that is valued by the voter. In that sense, our set-up captures a broad array of political considerations as the only substantive assumptions are that the voter values the politician's investment, that investment is costly to the politician, and the politician has a finite amount of resources at his disposal to take actions valued by the voter.

The interaction between the politician and the bureaucrat is also worth considering further. When we refer to the bureaucrat's "effort" or "output" and the politician "diverting" or "consuming" the bureaucrat's output, we have the following in mind. The bureaucrat invests in policy expertise initially, then the politician sets the policy agenda, where some of the policies align with the bureaucrat's own ideology or beliefs, while for others the bureaucrats are neutral or against. This can represent a bureaucrat committed to his agency's mission and dislikes his effort being directed toward policies or programs that do not further those goals. The main substantive assumptions simply denote a political setting in which there is a conflict of interest between the bureaucrat and the politician – the bureaucrat wants his effort to go towards goals that he values while the politician would like to utilize the expertise developed by the bureaucrat for his own ends that are not valued by the bureaucrat. This further implies that the bureaucrat has stronger incentives to invest in expertise if

he expects that the politician's agenda is more aligned with his own intrinsic motivations.[12] This is the major political dynamic we sought to capture in the model.

Finally, the assumption that the voter's reelection decision depends only on what the politician provides in terms of public goods is, in a sense, without loss of generality. While reelection could, in theory, depend on both the sharing rule and bureaucratic effort, for our main result this would not make a difference. In particular, we can have that reelection depends on two thresholds, one for public goods and one for bureaucratic effort. In equilibrium,[13] these two thresholds would be independent. The main result–the comparative statics between the no bureaucrat benchmark and the bureaucrat case–is applicable only for the threshold regarding the provision of public goods, and the conclusion would continue to hold in this case.

3 Benchmark: pure electoral accountability

Before turning to analysis of the full model we establish benchmark results where there is no bureaucrat. This allows us to first explore how electoral accountability, vis-á-vis the voter's reelection threshold, impacts the politician's public goods provision incentives without having to worry about the additional accountability relationship between the elected official and the bureaucrat. We then analyze the full model with all three players and compare the results to those from this benchmark case. This allows us to provide insight into how explicitly taking into account multiple layers of accountability structures voters' ability to hold elected representatives to high reelection standards.

In the benchmark model there are only two relevant choices. The politician chooses how much to invest in public goods given the budget, $g_t \leq b_t$, and the voter chooses to retain the politician or not once she observes g_t. Ultimately, our focus is on comparing the voter's optimal reelection rule from this benchmark model, γ_B^*, to the optimal reelection rule from the full model, γ_F^*, to explore how formally considering the politician-bureaucrat relationship in the governing hierarchy structures the voter's incentives for holding the politician to more or less stringent accountability standards.

To begin the analysis, consider the politician's public goods provision strategy. Budgets can

[12] This is consistent with previous work on bureaucratic motivations (Gailmard and Patty 2007; Prendergast 2007).

[13] One that is subject to stationarity restrictions and a welfare criterion we shall discuss further below.

be realized that make it impossible for the politician to produce enough public goods to satisfy the voter. Formally, given a voter reelection rule γ, when $b < \gamma$ the politician can not produce public goods sufficient to be reelected. In such a scenario, the politician diverts the entire budget for his own private benefit since any public goods produced harm his own welfare without any concomitant improvement in reelection prospects. When $b \geq \gamma$ the politician can produce public goods sufficient to be reelected. Specifically, so long as $g \geq \gamma$ the voter will reelect. There is no reason for the politician to ever invest more than is necessary to stay in office since his utility is strictly increasing in the amount of the budget he is able to divert away from public goods towards his own ends. Thus, when $b \geq \gamma$ the politician decides between allocating just enough of the budget toward public goods to remain in office, $g = \gamma$, given his continuation value of doing so, or diverting the entire budget toward private goods today but foregoing reelection. Letting V denote the politician's continuation value for remaining in office, the incentive compatibility constraint that must be satisfied to justify investing $g = \gamma$ to be reelected is given by,

$$\begin{aligned} b - \gamma + \delta V &\geq b, \\ \delta V &\geq \gamma. \end{aligned} \tag{1}$$

So long as the politician's continuation value for remaining in office is larger than the minimum level of public goods the voter will accept for reelection the politician will invest γ of the budget toward public goods, be subsequently reelected, and divert the rest of the budget for his own consumption. In contrast, if Inequality (1) fails to hold – i.e., if the level of public goods required for reelection is too demanding relative to the benefits of remaining in office – then the politician will again divert the entire budget for private purposes and accept being replaced by the voter.

The politician's continuation value, V, consists of his value from private budget consumption today plus the value of consumption in the future. Further, his expected value today consists of two components. First, there is the expected value when the budget is realized too low so that he cannot satisfy the voter's threshold for public goods: $b < \gamma$. Second, there is the expected value if the

budget is large enough to allow him to produce enough public goods to remain in office: $b \geq \gamma$. In the former case, the politician only gets the budget today because he loses the election. In the latter case, the politician gets part of the budget today and the value from remaining in office in the next period. Accordingly, the politician's continuation value satisfies the following:

$$\begin{aligned} V &= \int_0^{\gamma} b dF + \int_{\gamma}^{\infty} b - \gamma + \delta V dF, \\ &= \mathbb{E}[b] + (1 - F(\gamma))(\delta V - \gamma) \end{aligned}$$

Solving for V we obtain,[14]

$$V = \frac{\mathbb{E}[b] - (1 - F(\gamma))\gamma}{1 - (1 - F(\gamma))\delta}, \tag{2}$$

where $1 - F(\gamma)$ is the probability the budget is larger than the reelection threshold: $b \geq \gamma$. With the politician's best response to a generic reelection threshold γ in hand we can now turn to analyzing the voter's optimal reelection threshold.

3.1 Electoral accountability when there is no bureaucrat

The voter sets her optimal reelection threshold taking into account the politician's incentives to invest in public goods. A threshold that is too demanding increases the chance that the budget will be realized too low, which leads the politician to produce no public goods and instead divert the entire budget toward private benefits. On the other hand, a reelection threshold that is too lax leaves public goods on the table that the politician would have produced to remain in office. Accordingly, the voter sets her optimal reelection rule γ_B^* that maximizes her single-period payoff while also taking into account the politician's incentives for public goods provision:

$$\gamma_B^* = \max_{\gamma} \ (1 - F(\gamma))\gamma \ \text{ such that } \delta V \geq \gamma \tag{3}$$

When the constraint in Equation (3) does not bind it is always incentive compatible for the politician to provide sufficient public goods to be reelected, $g = \gamma$. Thus, the optimal reelection

[14] Note that V is strictly positive because $\mathbb{E}[b] > (1 - F(\gamma))\gamma$.

threshold is simply the unconstrained solution to the voter's maximization problem, which we denote as γ_B^U:

$$\gamma_B^U = \frac{1 - F(\gamma_B^U)}{f(\gamma_B^U)}. \tag{4}$$

Our assumption that F is log-concave implies that γ_B^U is monotone decreasing, which further implies that γ_B^U is unique. Alternatively, the politician's incentive compatibility constraint may bind. In this case the optimal reelection threshold is the constrained solution, which we denote as $\overline{\gamma}_B \coloneqq \gamma$ such that $\delta V = \gamma$, and is given by:

$$\overline{\gamma}_B \quad = \quad \delta \mathbb{E}[b]. \tag{5}$$

That is, $\overline{\gamma}_B$ is the upper bound for unconstrained reelection thresholds in the benchmark model, which is equivalent to the constrained reelection threshold. The preceding analysis yields the following result, which characterizes the optimal reelection threshold in the benchmark model given politician best responses.

Lemma 1. *In the benchmark model, the range of reelection thresholds γ that satisfy the politician's incentive compatibility constraint to positively invest in public goods (conditional on $b_t \geq \gamma$) is an interval $[0, \overline{\gamma}_B]$ where $\overline{\gamma}_B = \delta \mathbb{E}[b]$ is the reelection threshold that solves the politician's incentive compatibility constraint with equality: $\overline{\gamma}_B \coloneqq \gamma$ such that $\delta V = \gamma$.*

When the voter's optimal reelection threshold is interior to $[0, \overline{\gamma}_B]$ she can simply maximize her per-period utility by setting γ_B^U. Otherwise, when the politician's incentives for public goods provision are binding she does the best she can by setting $\overline{\gamma}_B$. This, in expectation, balances the voter's desire to induce the politician to provide more public goods and the risk that she is too demanding, in which case the politician foregoes reelection altogether and diverts the entire budget toward private goods.

Moreover, as players become more patient and the magnitude of the expected budget increases $\overline{\gamma}_B$ increases and the range of potential optimal reelection thresholds that are unconstrained by politician incentives expands. This makes it relatively more likely that the voter's optimal re-

election threshold will be the unconstrained solution to her maximization problem in Equation (3). Conversely, as players become more myopic and/or the expected budget decreases it is more likely that the politician's incentive compatibility constraint for public goods provision will bind and the voter's optimal reelection threshold will be constrained, $\gamma_B^* = \overline{\gamma}_B$. In line with these intuitions the following result provides conditions under which the voter's optimal reelection threshold in the benchmark model will be unconstrained, or constrained, by the politician's incentives to provide public goods.

Proposition 1. *Suppose* $\mathbb{E}[b] > \gamma_B^U$. *Then there exists a threshold* $\overline{\delta}$ *such that if* $\delta > \overline{\delta}$ *then the voter's reelection strategy that maximizes her welfare is the unconstrained solution to Equation* (3)*:* $\gamma_B^* = \gamma_B^U = \frac{1-F(\gamma_B^U)}{f(\gamma_B^U)}$. *In contrast, if* $\mathbb{E}[b] > \gamma_B^U$ *but* $\delta < \overline{\delta}$ *or* $\mathbb{E}[b] < \gamma_B^U$ *then the voter's reelection strategy that maximizes her welfare is the constrained solution to Equation* (3)*:* $\gamma_B^* = \overline{\gamma}_B = \delta\mathbb{E}[b]$.

Proposition 1 characterizes the environments in which the voter's optimal reelection threshold is constrained by the politician's incentives or not. The politician faces two constraints, both of which affect the voter's calculus. The first is a direct constraint in which the politician cannot produce enough public goods to be reelected because the realized budget is too small relative to the voter's demands. The second is an incentive constraint in which the politician, assuming the realized budget is large enough to allow for sufficient public goods provision, may benefit from diverting all of the budget today even though that means he will lose office. When the expected budget is sufficiently large and players are sufficiently patient, i.e., $\mathbb{E}[b] > \gamma_B^U$ and $\delta > \overline{\delta}$, the voter need only concern herself with the first, direct constraint. That is, she sets her optimal reelection threshold to balance the amount of public goods produced, given that the *realized* budget is large enough, with the risk that the realized budget turns out to be insufficient. She need not worry about the politician's incentive constraint because a sufficiently patient politician who expects to receive large budgets into the future also expects that being reelected will prove lucrative since he will be able to continue to extract large rents.

However, if either of the players are not sufficiently patient or the expected budget is too small then the voter must also take into account the politician's incentive constraint. If the expected

budget is still large but the players are impatient then the voter must deal with the fact that the politician finds diversion of the budget today more attractive compared to staying in office and extracting (higher) discounted rents. Similarly, if the expected budget is relatively small $\left(\text{i.e., } \mathbb{E}[b] < \gamma_B^U\right)$ then the likelihood that the budget will provide sufficient rents for the politician into the future is small. Even if the realized budget today is large enough to satisfy the voter it is likely that it will not be tomorrow and therefore the incentive to divert the entire budget today is strengthened. Moreover, the politician's incentives to divert the entire budget today are increasing as he becomes more myopic and as the expected budget decreases. In these scenarios, the voter finds herself constrained by the politician's incentives and does the best she can by setting $\gamma^* = \overline{\gamma}_B$. Note that the optimal threshold when the constraint binds is lower than when the constraint does not bind.

Overall, in the benchmark model the politician weighs diverting the entire budget today versus satisfying the voter and remaining in office into the future while the voter attempts to set reelection thresholds that maximize public goods provision. In some scenarios the voter must more heavily weight the politician's incentives in her decision-making, while in others she can act without worrying since the politician's incentives skew toward satisfying her demands. In the next section we introduce a second layer of governmental policymaking by explicitly including a bureaucrat who is formally connected to the politician and only indirectly connected to the voter through the politician.

4 Accountability in the governing hierarchy

In this section we analyze the full model that includes politician-bureaucrat interactions and derive the voter's optimal reelection threshold in this setting. Our key comparisons deal with the voter's optimal reelection rule in light of the additional layer of accountability between the politician and bureaucrat. Before turning to the question of optimal electoral accountability in this environment, however, we first characterize equilibrium interactions between the politician and the bureaucrat.

4.1 The politician-bureaucrat relationship

Recall that in every period, the bureaucrat first invests effort to produce output and the politician chooses how much to expropriate for his own use. In order to incentivize effort from the bureaucrat,

which is beneficial to all parties, the politician must not divert all of the bureaucrats effort toward his preferred goals. In a one-shot setting, the politician cannot credibly promise to not do so; this is the classic hold-up problem. However, if the relationship is on-going then the politician can credibly commit to not redirecting all of the bureaucrat's output, thus resolving the hold-up problem. The long-term nature of the relationship means that one party can deter the other from reneging on promises by refusing to cooperate in the future (i.e., play trigger strategies). Thus, despite the lack of a formal contract between the politician and the bureaucrat, there is a relational contract in which both parties have the incentive to commit to promises to sustain long-run cooperation.

Bureaucratic output. If the politician is expected to expropriate all of the bureaucrat's output then, as described above, the bureaucrat's unambiguous best response is to invest nothing. Positive sharing ($\alpha > 0$) and positive bureaucratic output ($e(\alpha) > 0$) are interrelated. Moreover, the credibility of the politician promising $\alpha > 0$ depends heavily on the likelihood that he remains in office, which further depends on the probability that the budget is realized large enough to satisfy the voter's reelection threshold: i.e., $1 - F(\gamma)$. For the purposes of deriving the bureaucrat's optimal output consider a generic sharing rule $\alpha > 0$ and probability of reelection given a generic reelection threshold, $1 - F(\gamma)$, which also represents the probability that the politician follows through on the sharing rule. This can be conceptualized as fixing the terms of the politician-bureaucrat relational contract. That is, the bureaucrat expects that the politician will not expropriate α of his output e with probability $1 - F(\gamma)$, which can be thought of as the terms and strength of the relational contracting environment between the two players. Accordingly, for a given sharing rule α that is respected with probability $1 - F(\gamma)$, the bureaucrat invests e to solve the following maximization problem,

$$\max_{e} \ \alpha(1 - F(\gamma))e - \frac{e^2}{2\lambda}.$$

Solving for e yields the bureaucrat's optimal effort investment given α and $1 - F(\gamma)$:

$$e^*(\alpha) = \alpha(1 - F(\gamma))\lambda. \tag{6}$$

Equation (6) illustrates that for a given α, that is credible with probability $1-F(\gamma)$, the bureaucrat's optimal effort is increasing in the probability that the politician will follow through on his sharing rule, which also implies that effort incentives are strengthened as it becomes more likely that the politician will remain in office and the politician-bureaucrat relational contract will remain intact. Intuitively, the bureaucrat also invests more effort as the costs of that effort decrease (i.e., as λ increases). Overall, bureaucratic output is higher (1) the more the politician promises to share, (2) the more credible that promise is, and (3) the cheaper it is to produce output.

Politician behavior. The bureaucrat invests effort based, in part, on how much of it he is able to direct toward the goals he intrinsically values. The politician is willing to compensate the bureaucrat for this effort, rather than expropriate it all, only if there is a long-term relationship at stake. When allowing the bureaucrat the discretion of using α of his output for ends he values today leads the bureaucrat to also work hard tomorrow, the relationship can be sustained without an explicit contract.

To characterize when the politician will follow through on his promise of sharing rule α let W denote the politician's continuation value in the politician-bureaucrat subgame. Then, the politician will follow through with sharing rule α if and only if the following condition is satisfied,

$$\begin{aligned} (1-\alpha)e^*(\alpha)+\mathbb{I}\delta W &\geq e^*(\alpha), \\ \mathbb{I}\delta W &\geq \alpha e^*(\alpha). \end{aligned} \tag{7}$$

where $\mathbb{I}$ is the indicator function taking value one when the politician is reelected and zero otherwise, and $e^*(\alpha)=\alpha(1-F(\gamma))\lambda$ from Equation (6) in the previous section. Inequality (7) shows that in order for following through on the promise of sharing α of $e^*(\alpha)$ to be incentive compatible it must be the case that the politician's continuation value of sustaining a cooperative relationship with the bureaucrat into the future (δW) outweighs the cost of *not* simply expropriating the extra $\alpha e^*(\alpha)$ of the bureaucrat's output today. In general, it is more likely that Inequality (7) will be satisfied as players become more patient and as the politician's continuation value W increases. However, this depends crucially on the likelihood that the politician remains in office. If he will definitely

not be reelected (if $\mathbb{I} = 0$) then Inequality (7) can never be satisfied with any positive sharing rule $\alpha > 0$, which is consistent with the intuition that there is no reason for the politician to respect his relationship with the bureaucrat today if there will be no relationship tomorrow. Put another way, if there is no chance for the politician to be reelected then he would expropriate all bureaucratic output for his own purposes ($\alpha = 0$) because there is no prospect of a long-term relationship and therefore no reason to respect the relational contract.

When the politician will be reelected it is possible for the cooperative relationship to be sustained, but this depends on the politician's continuation value W. Assuming that Inequality (7) is satisfied when the politician expects to be reelected (i.e., $\mathbb{I} = 1$),[15] the politician's continuation value W satisfies:

$$W = F(\gamma)e^*(\alpha) + (1 - F(\gamma))\left[(1-\alpha)e^*(\alpha) + \delta W\right]. \tag{8}$$

With probability $F(\gamma)$ the realized budget will not be large enough to satisfy the voter's demands for public goods and the politician, in turn, knowing he will not be able to secure reelection, will expropriate all of the bureaucrat's output which yields him $e^*(\alpha)$. In contrast, with probability $1 - F(\gamma)$ the realized budget will be large enough so that the politician will invest $g = \gamma$ in public goods and secure reelection. Moreover, because he will be reelected the politician keeps his promise of sharing α, which implies that he expropriates $(1-\alpha)e^*(\alpha)$ of the bureaucrat's output and also receives his continuation value of remaining in office and interacting with the bureaucrat into the future (δW). Solving Equation (8) for W yields the politician's continuation value,

$$W = \frac{F(\gamma)e^*(\alpha) + (1 - F(\gamma))(1-\alpha)e^*(\alpha)}{1 - \delta(1 - F(\gamma))}. \tag{9}$$

Plugging Equation (9) into Inequality (7) and solving for α, assuming that $\mathbb{I} = 1$, yields the range of equilibrium sharing rules that can be sustained in equilibrium.[16] Specifically, it shows that the politician's incentive compatibility constraint to follow through on his promise of α is satisfied for

[15] Of course, even when $\mathbb{I} = 1$, if the politician's incentive compatibility constraint in (7) is not satisfied then he will expropriate all of the bureaucrat's output.

[16] See the proof of Proposition 2 in the Appendix for details.

all $\alpha \leq \lambda$.[17]

Taken together, all of the preceding analysis – the bureaucrat's optimal output and the politician's incentives to follow through on his promise – yields the following result, which characterizes politician-bureaucrat equilibrium in the full model.

Proposition 2. *Suppose the voter adopts a stationary reelection threshold γ. Then for $\alpha \in [0, \delta]$ there exists an equilibrium in which the bureaucrat invests $e^*(\alpha) = \alpha(1 - F(\gamma))\lambda$ in every period and the politician best responds by sharing α and consuming $1 - \alpha$ himself, so long as $b \geq \gamma$. If $b < \gamma$ then the politician diverts the entire budget for his own benefit, shares nothing with the bureaucrat ($\alpha = 0$), and the bureaucrat invests nothing ($e^*(0) = 0$).*

The equilibrium described in Proposition 2 relies on the grim-trigger strategy that was described at the beginning of this section. Any deviation by the politician induces an equilibrium whereby the politician expropriates all bureaucratic output and the bureaucrat in turn invests no effort.[18] This is also important for understanding the nature of the relational contract between the politician and the bureaucrat that underpins equilibrium. The grim-trigger strategies are akin to building a "reversion penalty" into the relational contract itself.

Overall, Proposition 2 shows that the bureaucrat's optimal effort investment depends on the sharing rule α, as it determines the returns he can expect from his effort and $1 - F(\gamma)$ which is the probability that the realization of the budget is large enough so that the politician can provide enough public goods to secure reelection. The bureaucrat takes into account the likelihood of the politician's reelection since in the event of $b < \gamma$ the politician expects to be voted out and therefore has no incentive to keep his promise to not expropriate all bureaucratic output. Accordingly, the bureaucrat's effort investment incentives are stronger the more likely he believes it is that the politician will remain in office and their relational contract will persist. It follows, then, that the accountability standard, in terms of public goods, that the voter imposes on the politician affects the bureaucrat's output by altering the probability that the incumbent will remain in office (i.e., that the realized bud-

[17] This follows from $\delta W \geq \alpha e^*(\alpha) \Rightarrow \delta \left(\frac{F(\gamma)\alpha(1-F(\gamma))\lambda + (1-F(\gamma))(1-\alpha)\alpha(1-F(\gamma))\lambda}{1-\delta(1-F(\gamma))} \right) \geq \alpha^2(1-F(\gamma))\lambda \Rightarrow \lambda \geq \alpha$.

[18] See Lemma 3 in the Appendix.

get is large enough to satisfy the voter's standard). Thus, in contrast to the baseline model where there is no politician-bureaucrat relationship to account for, the optimal reelection threshold from the voter's perspective now takes into account how it impacts the stability of the relationship between the politician and the bureaucrat.

4.2 The voter-politician relationship

In this section we analyze the optimal reelection threshold set by the voter given the politician-bureaucrat relationship analyzed above. We compare the optimal thresholds from this full model to the benchmark model across two political-economic environments. First, we focus on the environment in which the optimal threshold in the benchmark is the *unconstrained* solution to Equation (3) and, second, the setting in which the optimal no bureaucrat benchmark threshold is the *constrained* solution to Equation (3). Before turning to those comparisons, however, we first derive the analogous unconstrained and constrained reelection thresholds when a bureaucrat is included, and establish some results that help characterize equilibrium in the full model.

4.2.1 Electoral accountability with the full hierarchy

In contrast to the benchmark no-bureaucrat model, the voter now sets her optimal reelection threshold taking into account the interactions between the politician and the bureaucrat. She still balances the same trade-off with respect to politician incentives as before – thresholds that are too stringent run the risk of the politician diverting the entire budget, while those that are too lax leave public goods on the table. However, as shown above, the stringency of the voter's demands also impact the bureaucrat's incentives to invest effort in government output. Specifically, increasing the threshold decreases the probability the incumbent will have a budget sufficient to secure reelection, which also weakens the credibility of the politician's promise not to expropriate all of bureaucrat's output. In turn, the bureaucrat's incentives to invest in output are weakened as well. Accordingly, the voter sets her optimal reelection rule γ_F^* to maximize her single-period payoff while accounting for both

the politician's and the bureaucrat's incentives:

$$\gamma_F^* = \max_{\gamma} \ (1 - F(\gamma))\gamma + e^*(\alpha) \quad \text{such that } \delta(V+W) \geq \gamma + \alpha e^*(\alpha). \tag{10}$$

The first term in Equation (10) is the same as in the benchmark. This simply reflects the impact that γ has on the likelihood that the budget will be realized large enough that the politician will produce $g = \gamma$ public goods. In addition, the voter also takes into account the bureaucrat's equilibrium output $e^*(\alpha)$. Finally, the voter still must take into account the politician's overall incentives not to divert the entire budget and to sustain his cooperative relationship with the bureaucrat into the future. This requires that the politician's continuation values for (1) investing sufficiently in public goods to secure reelection (V) and (2) not expropriating all of the bureaucrat's output (W) outweigh what he could gain (on net) by instead diverting the entire budget (which yields him an extra γ) and expropriating all of the bureaucrat's output (which yields him an extra $\alpha e^*(\alpha)$).[19]

When the constraint does not bind the politician will always provide enough public goods to satisfy the voter: $g = \gamma$. In that case, the optimal reelection threshold is the unconstrained solution to Equation (10), which we denote analogous to the benchmark:

$$\gamma_F^U = \frac{1 - F\left(\gamma_F^U\right)}{f\left(\gamma_F^U\right)} - \alpha\lambda. \tag{11}$$

Again, the assumption that F is log-concave ensures that γ_F^U is monotone decreasing and, therefore, unique. If the politician's incentive compatibility constraint binds then the optimal reelection threshold is the constrained solution to Equation (10), which we denote as $\overline{gamma}_F \coloneqq \gamma$ such that $\delta(V+W) = \gamma + \alpha e^*(\alpha)$, and is given by:

$$\overline{\gamma}_F = \delta\mathbb{E}[b] + e^*(\alpha)\left(\alpha + \delta\left(1 - 2\alpha\left(1 - F\left(\overline{\gamma}_F\right)\right)\right)\right) \tag{12}$$

[19]The politician's overall continuation value is $V+W$ since his utility is separable in budget diversion and the share of bureaucratic output he expropriates. Moreover, the addition of the bureaucrat does not change the politician's continuation value from securing reelection by producing sufficient public goods V; the only difference is that he now has an additional incentive to continue in office while maintaining a cooperative relationship with the bureaucrat in order to expropriate output, which is captured by W.

Comparing the unconstrained and constrained reelection thresholds in the full model to those in the benchmark reveals that optimal electoral accountability changes when the full hierarchy – elected official and unelected bureaucrat – is considered simultaneously. First, the unconstrained reelection threshold in the full model is lower than in the benchmark whenever there is positive sharing ($\alpha > 0$): $\gamma_F^U < \gamma_B^U$.[20] Moreover, the less of the bureaucrat's output the politician expropriates (i.e., the higher is α) and the lower the marginal cost of effort investment (i.e., the larger is λ) the more lenient the voter's equilibrium demands for public goods become. Second, the constrained reelection threshold in the full model is higher than the constrained threshold in the benchmark as long as there is positive bureaucratic output in equilibrium ($e^*(\alpha) > 0$).[21] Further, $\overline{\gamma}_F$ is increasing in bureaucratic output which implies that the range of optimal unconstrained reelection thresholds is larger in the full model. That is, the politician's incentives as a constraint on the voter's optimal demand for public goods are more lax.[22] The following lemma formalizes the preceding analysis.

Lemma 2. *In the full model, the range of reelection thresholds γ that satisfy the politician's incentive compatibility constraint to invest in public goods (conditional on $b_t \geq \gamma$) is an interval $[0, \overline{\gamma}_F]$ where $\overline{\gamma}_F \geq \overline{\gamma}_B$. The inequality is strict ($\overline{\gamma}_F > \overline{\gamma}_B$) if the politician follows sharing rule $0 < \alpha < \delta$ in equilibrium.*

Lemma 2 establishes the range of potential non-trivial equilibrium reelection thresholds[23] in the full model and shows that this range – an interval analogous to the benchmark – is larger in the full model, as described above. In general, the range of non-trivial reelection thresholds is determined by the politician's continuation value from holding office. The higher his continuation value, the more willing the politician is to satisfy the reelection threshold. In the baseline case without the bureaucrat, the politician's continuation value depends only on the possibility of diverting part of the budget toward private goods. In the full game, the politician not only has the opportunity to divert budgetary resources, but also some of the bureaucratic output. It follows that there is a greater

[20] This also requires $\lambda > 0$, which is always satisfied by definition.

[21] As we saw in the previous section, positive sharing – $\alpha > 0$ – is important in promoting positive bureaucratic output.

[22] This can also be seen by noting that $W > \alpha e^*(\alpha)$ in the constraint on Equation (10), which implies the constraint is more lax than the analogous constraint in the benchmark.

[23] By non-trivial here we mean that the politician is willing to meet the threshold conditional on $b > \gamma$.

continuation value for holding office in the full hierarchy because of this secondary source of private rents, which further implies that the range of nontrivial equilibrium reelection thresholds is larger in the full game. The following corollary also follows from the arguments underpinning Lemma 2.

Corollary 1. *If the equilibrium reelection threshold in the benchmark is unconstrained, $\gamma_B^* = \gamma_B^U$, then the equilibrium threshold for the full game is also unconstrained, $\gamma_F^* = \gamma_F^U$.*

Recall that the voter chooses γ_B^* to maximize her welfare subject to the politician's incentive compatibility constraint to invest positively in public goods. Given that this constraint is more relaxed in the full game – as documented in Lemma 2 – if the constraint does not bind for the voter in the baseline case, then it also does not bind in the full model. Thus, if the optimal reelection threshold was unconstrained in the benchmark case with no bureaucrat, then it is also unconstrained in the full model with the bureaucrat. This follows from the intuition that now that the politician has stronger incentive to remain in office to continue to collect rents from the bureaucrat the need for the voter to account for those politician incentives is weaker (i.e., the constraint on the voter's optimal threshold choice is more lax). We now turn to comparing the optimal electoral threshold in the full model to those from the benchmark based on the nature of the political-economic environment.

Favorable political-economic environment. Proposition 1 provides conditions under which the politician's constraint does not bind for the voter's problem in the benchmark case. Specifically, as long as the players are sufficiently patient and expect that the budget will be large then the voter maximizes her own welfare without worrying about the politician's incentives. It follows from Corollary 1 that these same conditions apply for when the voter's optimal reelection threshold is unconstrained in the full game. Proposition 3 shows that the voter optimal threshold for the politician in the full game is *lower* than in the baseline case, implying that the voter weakens the standards to which she holds the politician accountable.

Proposition 3. *Suppose $\mathbb{E}[b] > \gamma_B^U$ and $\delta > \overline{\delta}$ so that the equilibrium reelection threshold in the benchmark is $\gamma_B^* = \gamma_B^U$. Then the voter welfare maximizing reelection threshold in the full game, denoted by $\gamma_F^* = \gamma_F^U$, is strictly less than γ_B^* from the benchmark: $\gamma_F^* < \gamma_B^*$.*

In the benchmark case without the bureaucrat, the voter considers only how the reelection threshold she sets impacts the politician's performance. In particular, there is a trade-off between the level of public goods the voter demands and the probability that the politician will invest sufficiently in public goods to satisfy those demands. This trade-off is optimized, i.e., marginal benefit equaling marginal cost, when the politician's incentive compatibility constraint is not binding. In the full game, on the other hand, the reelection threshold not only matters for the politician's incentives but also indirectly for the bureaucrat's incentives. Specifically, as discussed following Proposition 2, the bureaucrat's effort level is *increasing* in the probability of the politician's reelection. Thus, when the voter is unconstrained in the sense of not needing to accommodate the politician's incentives she can instead focus on strengthening the bureaucrat's effort incentives.

To do so, she weakens her demand for public goods from the politician by lowering her reelection threshold (relative to the benchmark), which increases the probability that the politician remains in office by increasing the likelihood that the realized budget will be sufficient to satisfy her threshold. As discussed above, this has two interrelated effects that jointly lead to higher output from the bureaucrat. First, the politician's incentives to renege on the relational contract with the bureaucrat are weakened because his tenure in office is more secure, which increases the value of cooperation with the bureaucrat into the future. Second, since the politician is now less likely to renege on his promise of α the bureaucrat has stronger incentives to invest more effort toward government output. Simply put, the voter is able to strengthen the relational contract between the politician and the bureaucrat by lessening her public goods demands on the politician, which benefits her due to the increased output this induces from the bureaucrat. This is true even though there is no direct connection between the voter and the bureaucrat: the effect feeds indirectly through the voter's ability to manipulate the probability that the incumbent remains in office. Finally, recall that this benefit comes without additional cost since in this setting she does not have to incorporate the politician's public goods incentives (i.e., the politician's constraint does not bind).

Unfavorable political-economic environment. We have so far seen that the voter weakens her demands on the politician by lowering her reelection threshold to account for the bureaucrat's in-

centives when the political-economic environment is such that she does not have to concern herself with the politician's incentives – when the players are sufficiently patient and the expected budget is sufficiently large. When the environment is such that the voter is constrained in the benchmark – i.e., when either the expected budget is too small or players are relatively myopic – the effect of adding the bureaucratic layer of accountability on electoral accountability is ambiguous. Proposition 4 characterizes equilibrium electoral accountability in this setting.

Proposition 4. *Suppose that either* $\mathbb{E}[b] < \gamma_B^U$ *or* $\mathbb{E}[b] > \gamma_B^U$ *and* $\delta < \overline{\delta}$ *so that the equilibrium reelection threshold in the benchmark model is* $\gamma_B^* = \overline{\gamma}_B$*. Then, in the full model:*

- *If* $\gamma_F^U > \gamma_B^*$ *then the equilibrium reelection threshold* γ_F^* *is greater than* γ_B^* *so that the voter is more demanding in the full model compared to the benchmark.*

- *If* $\gamma_F^U < \gamma_B^*$ *then the equilibrium reelection threshold is* $\gamma_F^* = \gamma_F^U$ *and is less than* γ_B^* *so that the voter is less demanding in the full model compared to the benchmark.*

When expected budgets are relatively small and players are relatively myopic – the voter can benefit from sometimes tightening her leash on the politician while at other times she benefits from loosening it (as in the unconstrained case above), as compared to the benchmark setting in which she does not need to account for the politician-bureaucrat relationship. In general, the presence of the bureaucrat has two potential effects on the voter's calculus. First, the voter has stronger incentive to promote stability in terms of the politician's tenure in office since this translates into higher bureaucratic output through strengthening the politician-bureaucrat relational contract. This lends itself to weakening her demands for public goods. Second, the bureaucrat's presence relaxes the constraint on the voter's maximization problem because now the politician has a higher continuation value for staying in office. This second effect comes into play when, in the benchmark case, the voter was constrained and would have rather demanded more in the way of public goods. Now that the bureaucrat is present in the hierarchy and the range of optimal electoral thresholds is expanded the voter has the opportunity to increase her demands on the politician.[24]

[24]Note that this second effect is not applicable in the previous case in which the voter is unconstrained in both the benchmark and the full hierarchy model.

If the voter was severely constrained when there was no bureaucrat – $\gamma_F^U > \gamma_B^* = \overline{\gamma}_B$ – then the second effect dominates. The environment is such that the voter would have preferred, in the benchmark, to demand more but could not due to the politician's incentives. Now that the presence of the bureaucrat has increased the value of remaining in office for the politician the voter is no longer as severely constrained and can be more demanding in the level of public goods she requires the politician to produce in order to secure reelection. This dynamic applies whether or not the equilibrium threshold in the full hierarchy model is unconstrained by the politician's incentives or not; in either case the voter can increase the threshold compared to the benchmark. This captures the intuition driving the first case in Proposition 4. The presence of a bureaucrat makes optimal electoral accountability more stringent than in a setting in which the voter need only worry about her elected representative in isolation from other governmental actors.

If instead the voter was not as strongly constrained in the benchmark when there was no bureaucrat – i.e., $\gamma_F^U < \gamma_B^* = \overline{\gamma}_B$ – then she is unconstrained in the full hierarchy model. This follows from the fact that the range of possible reelection thresholds that are unconstrained in the full hierarchy is larger than in the benchmark. Thus, if the voter was constrained in the benchmark but her unconstrained optimal threshold is less than that she need not worry about the politician's incentives and can instead weaken her demands on the politician in order to promote higher bureaucratic output, as in the previous section. That is, in the second case of Proposition 4 the first effect from above dominates – the voter benefits from promoting stability in the tenure of the politician, which strengthens the politician-bureaucrat relational contract and induces higher bureaucratic output. The presence of the bureaucrat again leads to optimal electoral accountability being more lax than a setting in which the politician-bureaucrat relationship does not matter.

5 Conclusion

In this paper we developed a model of hierarchical accountability with a voter, an elected representative, and a bureaucrat. The politician is both an agent, to the voter, and a principal, to the bureaucrat. Even though the voter has no direct role in overseeing the (unelected) bureaucrat, op-

timal electoral accountability is affected by the presence of the politician-bureaucrat relationship in the governing hierarchy. The need to promote cooperation between elected representatives and unelected bureaucrats when their interests conflict alters the normative desirability of more or less stringent electoral accountability standards. Relative to standard models of electoral accountability in which voter-politician or politician-bureaucrat relationships are analyzed in isolation from one another we provide results that show that these relationships are interrelated in important ways and, depending on the nature of the political-economic environment, can alter the optimal performance standards for which politicians should ultimately be held to account.

In particular, when the political-economic environment is relatively favorable to effective governance – expected budgets are large and players are farsighted – voters benefit from weakening their demands for politician performance in order to promote a long-term cooperative relationship between their elected representatives and unelected bureaucrats. When the political-economic environment is relatively more challenging – expected budgets are smaller and players are myopic – the additional consideration of politician-bureaucrat cooperation can cut both ways. One one hand, it can lead to more stringent accountability standards by relaxing the constraints on voter demands by increasing the value of holding office for politicians. On the other hand, it can also weaken accountability standards by increasing the relative value of politician-bureaucrat cooperation, similar to a more advantageous political-economic environment. These insights suggest that to better understand the subtleties of democratic accountability it is important to consider the myriad relationships that, both directly and indirectly, affect government performance. We provide one step toward understanding these complex accountability relationships inherent in modern governing hierarchies.

References

Ashworth, Scott. 2012. "Electoral Accountability: Recent Theoretical and Empirical Work." *Annual Review of Political Science* 15:183–201.

Ashworth, Scott and Ethan Bueno de Mesquita. 2006. "Delivering the Goods: Legislative Particularism in Different Electoral and Institutional Settings." *The Journal of Politics* 68(1):168–179.

Baker, George, Robert Gibbons and Kevin J Murphy. 2002. "Relational Contracts and the Theory of the Firm." *The Quarterly Journal of Economics* 117(1):39–84.

Barro, Robert J. 1973. "The Control of Politicians: An Economic Model." *Public choice* 14(1):19–42.

Besley, Timothy. 2006. *Principled Agents? The Political Economy of Good Government*. London: Oxford University Press.

Canes-Wrone, Brandice, Michael C. Herron and Kenneth W. Shotts. 2001. "Leadership and Pandering: A Theory of Executive Policymaking." *American Journal of Political Science* 45(3):532–550.

Daley, Brendan and Erik Snowberg. 2009. "Even if it is not Bribery: The Case for Campaign Finance Reform." *The Journal of Law, Economics, & Organization* 27(2):324–349.

Duggan, John and César Martinelli. 2017. "The Political Economy of Dynamic Elections: Accountability, Commitment, and Responsiveness." *Journal of Economic Literature* 55(3):916–84.

Ellis, Christopher and Thomas Groll. 2019. "Whom to Lobby? Special Interests and Hired Guns." *Unpublished manuscript. Columbia University* .
URL: *http://www.columbia.edu/ tg2451/research/Who%20lobbies%20whom_online.pdf*

Epstein, David and Sharyn O'Halloran. 1994. "Administrative Procedures, Information, and Agency Discretion." *American Journal of Political Science* 38(3):697–722.

Epstein, David and Sharyn O'Halloran. 1999. *Delegating Powers: A Transaction Cost Politics Approach to Policy Making Under Separate Powers*. Cambridge University Press.

Ferejohn, John. 1986. "Incumbent Performance and Electoral Control." *Public Choice* 50(1):5–25.

Fiorina, Morris P. 1981. *Retrospective Voting in American National Elections*. New Haven: Yale University Press.

Forand, Jean Guillaume. 2018. "Civil Service and the Growth of Government." *Unpublished manuscript. University of Waterloo* .
URL: *http://arts.uwaterloo.ca/~jgforand/Papiers/CivilService13.pdf*

Fox, Justin and Richard Van Weelden. 2012. "Costly Transparency." *Journal of Public Economics* 96(1-2):142–150.

Fox, Justin and Stuart V Jordan. 2011. "Delegation and Accountability." *The Journal of Politics* 73(3):831–844.

Gailmard, Sean. 2002. "Expertise, Subversion, and Bureaucratic Discretion." *Journal of Law, Economics, & Organization* 18(2):536–555.

Gailmard, Sean and John W Patty. 2007. "Slackers and Zealots: Civil Service, Policy Discretion, and Bureaucratic Expertise." *American Journal of Political Science* 51(4):873–889.

Gailmard, Sean and John W Patty. 2012. "Formal Models of Bureaucracy." *Annual Review of Political Science* 15:353–377.

Gersen, Jacob E. and Matthew C. Stephenson. 2014. "Over-accountability." *Journal of Legal Analysis* 6(2):185–243.

Halac, Marina. 2012. "Relational Contracts and the Value of Relationships." *American Economic Review* 102(2):750–79.

Hirsch, Alexander V. and Jonathan P. Kastellec. 2019. "A Theory of Policy Sabotage." *Unpublished manuscript, California Institute of Technology* .
URL: *https://www.its.caltech.edu/~avhirsch/hirsch_kastellec_policy_sabotage_for_web.pdf*

Hirsch, Alexander V. and Kenneth W. Shotts. 2012. "Policy Specific Information and Informal Agenda Power." *American Journal of Political Science* 56(1):67–83.

Hirsch, Alexander V. and Kenneth W. Shotts. 2015. "Competitive Policy Development." *American Economic Review* 105(4):1646–1664.

Hirsch, Alexander V. and Kenneth W. Shotts. 2018. "Policy-Development Monopolies: Adverse Consequences and Institutional Responses." *Journal of Politics* 80(4):1339–1354.

Huber, John and Charles Shipan. 2002. *Deliberate Discretion? The Institutional Foundations of Bureaucratic Autonomy*. New York, NY: Cambridge University Press.

Kartik, Navin and Richard Van Weelden. 2019. "Informative Cheap Talk in Elections." *Review of Economic Studies* 86(2):755–784.

Key, V.O. 1966. *The Responsible Electorate: Rationality in Presidential Voting, 1936-1960*. Cambridge, MA: Belknap.

Levin, Jonathan. 2003. "Relational Incentive Contracts." *American Economic Review* 93(3):835–857.

Li, Christopher M. 2019. "Indirect Accountability of Political Appointees." *Unpublished manuscript. Florida State University* .
URL: *https://drive.google.com/file/d/1gNRS0t3c3dSl37CIqYyAlkioig1-Ufe7/view*

MacLeod, W. Bentley and James M. Malcomson. 1989. "Implicit Contracts, Incentive Compatibility, and Involuntary Unemployment." *Econometrica* pp. 447–480.

Manin, Bernard. 1997. *Principles of Representative Government*. Cambridge: Cambridge University Press.

Maskin, Eric and Jean Tirole. 2004. "The Politician and the Judge: Accountability in Government." *American Economic Review* 94(4):1034–1054.

McCubbins, Mathew D, Roger G Noll and Barry R Weingast. 1987. "Administrative Procedures as Instruments of Political Control." *Journal of Law, Economics, & Organization* 3:243.

McCubbins, Matthew D, Roger G Noll and Barry R Weingast. 1989. "Structure and Process, Politics and Policy: Administrative Arrangements and the Political Control of Agencies." *Virginia Law Review* 75:431.

Miller, Gary J. 2005. "The Political Evolution of Principal-Agent Models." *Annual Review of Political Science* 8:203–225.

Moe, Terry M. 2012. "Delegation, Control, and the Study of Public Bureaucracy." *The Forum* 10(2):1–45.

Morelli, Massimo and Richard Van Weelden. 2013. "Ideology and Information in Policymaking." *Journal of Theoretical Politics* 25(3):412–439.

Patty, John W. and Ian R. Turner. 2019. "Ex Post Review and Expert Policymaking: When Does Oversight Reduce Accountability?" *Journal of Politics* Forthcoming.

Prendergast, Canice. 2007. "The Motivation and Bias of Bureaucrats." *American Economic Review* 97(1):180–196.

Sasso, Greg. 2019. "Delegation and Political Turnover." *Unpublished manuscript. Bocconi University* .
URL: *https://gregsasso.files.wordpress.com/2019/01/delegation-and-political-turnover-01-22-2019.pdf*

Ting, Michael M. 2009. "Organizational Capacity." *Journal of Law, Economics, & Organization* 27(2):245–271.

Turner, Ian R. 2017. "Working Smart *and* Hard? Agency Effort, Judicial Review, and Policy Precision." *Journal of Theoretical Politics* 29(1):69–96.

Turner, Ian R. 2019*a*. "Political Agency, Oversight, and Bias: The Instrumental Value of Politicized Policymaking." *Journal of Law, Economics, & Organization* .

Turner, Ian R. 2019*b*. "Reviewing Procedure vs. Judging Substance: The Scope of Review and Bureaucratic Policymaking." *Unpublished manuscript. Yale University* .
URL: *http://www.ianrturner.com/ONLINE-proc-sub-8-2019.pdf*

Ujhelyi, Gergely. 2014. "Civil Service Reform." *Journal of Public Economics* 118:15–25.

Vlaicu, Razvan and Alexander Whalley. 2016. "Hierarchical Accountability in Government." *Journal of Public Economics* 134:85–99.

Yazaki, Yukihiro. 2018. "The Effects of Bureaucracy on Political Accountability and Electoral Selection." *European Journal of Political Economy* 51:57–68.

A Supplemental Appendix

Notation	Description
γ_B^*	Equilibrium voter reelection threshold in the benchmark model
$\overline{\gamma}_B$	Upper bound for the unconstrained reelection threshold in the benchmark model and the constrained reelection threshold in the benchmark model
γ_B^U	Unconstrained reelection threshold in the benchmark model
γ_F^*	Equilibrium voter reelection threshold in the full model
$\overline{\gamma}_F$	Upper bound for the unconstrained reelection threshold in the full model and the constrained reelection threshold in the full model
γ_F^U	Unconstrained reelection threshold in the full model

Table 1: Voter reelection threshold notation

Table 1 provides a reference for the different electoral thresholds across the benchmark and full hierarchy models.

A.1 Benchmark model

Lemma 1. *In the benchmark model, the range of reelection thresholds γ that satisfy the politician's incentive compatibility constraint to positively invest in public goods (conditional on $b_t \geq \gamma$) is an interval $[0, \overline{\gamma}_B]$ where $\overline{\gamma}_B = \delta \mathbb{E}[b]$.*

Proof of Lemma 1. Note that the politician's IC constraint for meeting the public goods threshold γ is

$$\delta V \geq \gamma \tag{13}$$

where V stands for the politician's continuation value for staying in office. We now argue that the continuation value V is decreasing in γ. Specifically, writing $V(\gamma)$ explicitly as a function of γ, we want to show that $V(\gamma') \leq V(\gamma)$ for $\gamma' > \gamma$.

Let $\tau^* : \text{supp}F \to \mathbb{R}$ be the optimal policy that induces $V(\gamma')$ (a policy describes, for every realization of b, how much the politician takes as consumption for himself). Now notice that τ^* together with F implies a distribution over consumption paths (a consumption path is a history of what the politician consumes while in office). For a given a sequence of realized budget, let's compare the consumption path under γ vs γ' as induced by the policy τ^*. We claim that the consumption path under γ is at least as long as γ' and is equal for dates when the politician is in office under both scenarios. The reason is that politician can always consume $\tau^*(b)$ under γ. Moreover, whenever he remains in office under γ' (i.e., $\tau^*(b) \geq \gamma'$), he will remain in office under γ (i.e., $\tau^*(b) \geq \gamma$). This means that, facing threshold γ, the politician achieves a continuation value at least $V(\gamma')$ by using policy τ^*. Thus, the continuation value $V(\gamma)$, which is induced by the optimal policy, must be weakly greater than $V(\gamma')$. Therefore, the set of γ that satisfies the IC constraint $\delta V \geq \gamma$ must be an interval.

Now, simple calculation shows that the upper bound of the interval satisfies the following,

$$\overline{\gamma} \;=\; \delta\left(\frac{\mathbb{E}[b] - (1 - F(\overline{\gamma}))\overline{\gamma}}{1 - (1 - F(\overline{\gamma}))\delta}\right),$$

Rearranging and solving for $\overline{\gamma}$,

$$\begin{aligned}
\overline{\gamma} &= \frac{\delta\mathbb{E}[b]-\delta\overline{\gamma}+\delta F(\overline{\gamma})\overline{\gamma}}{1-\delta+\delta F(\overline{\gamma})}, \\
\overline{\gamma}-\delta\overline{\gamma}+\delta F(\overline{\gamma})\overline{\gamma} &= \delta\mathbb{E}[b]-\delta\overline{\gamma}+\delta F(\overline{\gamma})\overline{\gamma}, \\
\overline{\gamma} &= \delta\mathbb{E}[b], \qquad (14)
\end{aligned}$$

yields the upper bound as stated in the result. ■

Proposition 1. *Suppose $\mathbb{E}[b] > \gamma_B^U$. Then there exists a threshold $\overline{\delta}$ such that if $\delta > \overline{\delta}$ then the voter's reelection strategy that maximizes her welfare is the unconstrained solution to Equation* (3)*: $\gamma_B^* = \gamma_B^U = \frac{1-F(\gamma_B^U)}{f(\gamma_B^U)}$. In contrast, if $\mathbb{E}[b] > \gamma_B^U$ but $\delta < \overline{\delta}$ or $\mathbb{E}[b] < \gamma_B^U$ then the voter's reelection strategy that maximizes her welfare is the constrained solution to Equation* (3)*: $\gamma_B^* = \overline{\gamma}_B = \delta\mathbb{E}[b]$.*

Proof of Proposition 1. Recall that $\gamma_B^U = \text{argmax}(1-F(\gamma))\gamma$ (i.e., it is the solution to the unconstrained problem). It can be verified that the objective function is concave, and the FOC is

$$\gamma_B^U = \frac{1-F(\gamma_B^U)}{f(\gamma_B^U)}. \qquad (15)$$

The condition that $\frac{1-F(\gamma)}{f(\gamma)}$ is decreasing ensures an unique solution exists.

Recall that the politician's IC constraint in the baseline is

$$\delta\left(\frac{E(b)-(1-F(\gamma))\gamma}{1-(1-F(\gamma))\delta}\right) > \gamma$$

If $\gamma_B^U < E(b)$, then we get that as $\delta \to 1$,

$$\delta\left(\frac{E(b)-(1-F(\gamma_B^U))\gamma_B^U}{1-(1-F(\gamma_B^U))\delta}\right) \to \frac{E(b)-(1-F(\gamma_B^U))\gamma_B^U}{F(\gamma_B^U)} > \frac{F(\gamma_B^U)E(b)}{F(\gamma_B^U)} = E(b) > \gamma_B^U.$$

(i.e., the IC constraint is satisfied under γ_B^U). Thus, there exists $\bar{\delta}$ such that $\delta \geq \bar{\delta}$ implies that the solution to the unconstrained problem γ_B^U can be taken as the full solution.

Now, since $\bar{\gamma}_B = \delta\mathbb{E}[b] < E[b]$, if either $\gamma_B^U < \mathbb{E}[b]$ or $\mathbb{E}[b] > \gamma_B^U$ but $\delta < \overline{\delta}$, the unconstrained

solution γ_B^U would violate the constraint. Given the concavity of the objective function, it must be that $\gamma_B^* = \bar{\gamma}_B$. ■

A.2 Full model

Lemma 3. *The politician consuming all the bureaucratic output ($\alpha_t = 0$) and the bureaucrat investing zero effort into output ($e_t = 0$) is a subgame perfect equilibrium of the politician-bureaucrat stage game.*

Proof of Lemma 3. We verify this is an equilibrium by checking one stage deviations are not profitable for neither player at any stage of play. The politician, expecting the bureaucrat to exert zero effort going forward, can do no better than consuming all bureaucratic output now. For the bureaucrat, expecting the politician to consume all his output, maximizes his utility exerting zero effort. ■

Proposition 2. *Suppose the voter adopts a stationary reelection threshold γ. Then for $\alpha \in [0, \delta]$ there exists an equilibrium in which the bureaucrat invests $e^*(\alpha) = \alpha(1 - F(\gamma))\lambda$ in every period and the politician best responds by sharing α and consuming $1 - \alpha$ himself, so long as $b \geq \gamma$. If $b < \gamma$ then the politician diverts the entire budget for his own benefit, shares nothing with the bureaucrat ($\alpha = 0$), and the bureaucrat invests nothing ($e^*(0) = 0$).*

Proof of Proposition 2. The equilibrium is constructed using stationary grim-trigger strategies (and therefore we omit the time subcript). The politicians and the bureaucrat are expected to follow the prescribed strategy (α and $(1 - F(\gamma))\lambda\alpha$); any deviation leads to a reversion to the equilibrium described in Lemma 3. We will verify this this is an equilibrium.

Let W be the politician's continuation value in the game with the bureaucrat, treating the interaction with the voter as exogenous for now (in particular, suppose the voter's threshold is γ and is stationary). The politician finds it optimal to following through with sharing rule α (i.e., the IC constraint) if and only if,

$$(1 - \alpha)\beta\lambda\alpha + \mathbb{I}\delta W \geq \beta\lambda\alpha \tag{16}$$

where $\mathbb{I}$ is the indicator function of the politician's reelection, and $\beta \equiv 1 - F(\gamma)$. Note that the right hand side reflects the fact that 1) if the politician were to deviate and consume more than promised, it is best he consume everything and 2)after deviation, the prescribed strategy for the bureaucrat is to exert zero effort. Now, it is straightforward to see that if the politician expects to be voted out (i.e., $\mathbb{I} = 0$, which occurs whenever $b < \gamma$.) the constraint is necessarily violated and thus the politician consumes all of the bureaucratic output. Now let $\mathbb{I} = 1$ and suppose that the IC constraint (16), then the continuation value W satisfies

$$W = (1-\beta)\beta\lambda\alpha + \beta^2(1-\alpha)\alpha\lambda + \beta\delta W$$

Solving the equation we get that

$$W = \frac{(1-\beta)\beta\lambda\alpha + \beta^2(1-\alpha)\alpha\lambda}{1-\beta\delta} \tag{17}$$

Now the sharing rule α that maximizes voter welfare (effort) is the greatest possible value of α that satisfies the constraint (16) given that $\mathbb{I} = 1$. Simple algebra shows that the constraint after W is substitute out is quadratic in α and the greatest α that solves the constraint at equality is δ.

Given the politician's and the voter's strategy, the bureaucrat maximizes his utility by choosing effort that maximizes $(1 - F(\gamma))\alpha e - \frac{e^2}{2\lambda}$. The solution to the maximization problem yields $e^* = (1 - F(\gamma))\lambda\alpha$. ■

Lemma 2. *In the full model, the range of reelection thresholds γ that satisfy the politician's incentive compatibility constraint to invest in public goods (conditional on $b_t \geq \gamma$) is an interval $[0, \overline{\gamma}_F]$ where $\overline{\gamma}_F \geq \overline{\gamma}_B$. The inequality is strict $(\overline{\gamma}_F > \overline{\gamma}_B)$ if the politician follows sharing rule $0 < \alpha < \delta$ in equilibrium.*

Proof of Lemma 2. If 1) the politician and the bureaucrat deviate from equilibrium or 2) the sharing rule in equilibrium is zero or the maximal sharing rule $\alpha = \delta$, then the politician obtains zero continuation value from her interaction with the bureaucrat. Thus, the politician's IC constraint for

providing public good would be the same as in the benchmark setting. If the equilibrium sharing rule α is in $(0,\delta)$, then the politician does gain some continuation value from her interaction with the bureaucrat. Now, the politician's IC constraint for providing public goods is

$$\delta(V+W) \geq \gamma + \alpha^2(1-F(\gamma))\lambda \tag{18}$$

where V is the continuation value in the baseline model. We now need to show that $\delta W - \alpha^2(1-F(\gamma))\lambda$ is decreasing in γ, which, together with the already proven fact that $\delta V - \gamma$ is decreasing, will prove that the set of γ that satisfies the above IC is an interval. For notational simplicity, let $\beta = 1-F(\gamma)$. We know from previous calculations that,

$$W = \frac{\lambda\beta\alpha - \lambda\beta^2\alpha^2}{1-\beta\delta}.$$

Rearranging we have that,

$$\delta W - \alpha^2\beta\lambda = \lambda(\delta\alpha - \alpha^2)\underbrace{\left(\frac{\beta}{1-\beta\delta}\right)}_{:=A}.$$

Recall that $\alpha \leq \alpha^* = \delta$ and therefore $\frac{dA}{d\beta} = \frac{1}{(1-\beta\delta)^2} > 0$ implies that $\delta W - \alpha^2\beta\lambda$ is increasing in β. Further, since β is a decreasing function of γ, the expression would be decreasing in γ. Thus, the set of γ that satisfies the above IC is an interval. Let $\overline{\gamma}_F$ be the upper-bound of the interval. It must be that $\overline{\gamma}_F > \overline{\gamma}_B$ since $\delta W > \alpha^2(1-F(\gamma))\lambda$, which follows from the fact that equilibrium α must yield the politician strictly positive continuation value. ■

Corollary 1. *If the equilibrium reelection threshold in the benchmark is unconstrained, $\gamma_B^* = \gamma_B^U$, then the equilibrium threshold for the full game is also unconstrained, $\gamma_F^* = \gamma_F^U$.*

Proof of Corollary 1. Follows directly from $\overline{\gamma}_F \geq \overline{\gamma}_B$, which is shown in the proof of Lemma 2. ■

Proposition 3. *Suppose $\mathbb{E}[b] > \gamma_B^U$ and $\delta > \overline{\delta}$ so that the equilibrium reelection threshold in the benchmark is $\gamma_B^* = \gamma_B^U$. Then the voter welfare maximizing reelection threshold in the full game,*

denoted by $\gamma_F^* = \gamma_F^U$, is strictly less than γ_B^* from the benchmark: $\gamma_F^* < \gamma_B^*$.

Proof of Proposition 3. Comparing the politician's IC for providing public goods in the baseline model (i.e., inequality 13) vs. the IC constraint in the full model (i.e., inequality 18), it is easy to see that the politician's IC constraint for providing public goods is more relaxed. More precisely, if providing g is incentive compatible for the politician in the baseline model, then it is incentive in the full game as well.

Now recall that the condition $\mathbb{E}[b] > \gamma_B^U$ and $\delta > \overline{\delta}$ ensure that the IC constraint for providing public goods does not bind in the baseline model. To prove our result, we will show that $\gamma_B^U < \gamma_F^U$. Note that the unconstrained problem in the full game is

$$\max_{\gamma} (1 - F(\gamma))(\gamma + \alpha\lambda)$$

where $1 - F(\gamma)$ is the probability that it is feasible for the politician to provide γ. It can be verified that the objective function above is concave, and the FOC for γ_F^U is

$$\gamma_F^U = \frac{1 - F(\gamma_F^U)}{f(\gamma_F^U)} - \alpha\lambda. \tag{19}$$

The condition that $\frac{1-F(b)}{f(b)}$ is decreasing ensures an unique solution, γ_F^U, exists. And comparing equation 19 with the FOC in the baseline model (equation 15), it follows that $\gamma_F^U = \gamma_F^* < \gamma_B^* = \gamma_B^U$. ■

Proposition 4. *Suppose that either $\mathbb{E}[b] < \gamma_B^U$ or $\mathbb{E}[b] > \gamma_B^U$ and $\delta < \overline{\delta}$ so that the equilibrium reelection threshold in the benchmark model is $\gamma_B^* = \overline{\gamma}_B = \delta\mathbb{E}[b]$. Then, in the full model:*

- *If $\gamma_F^U > \delta\mathbb{E}[b]$ then the equilibrium reelection threshold γ_F^* is greater than γ_B^* so that accountability is more demanding in the full model compared to the benchmark.*
- *If $\gamma_F^U < \delta\mathbb{E}[b]$ then the equilibrium reelection threshold is $\gamma_F^* = \gamma_F^U$ and is less than γ_B^* so that accountability is less demanding in the full model compared to the benchmark.*

Proof of Proposition 4. If the IC constraint binds for the welfare maximizing threshold in the baseline model i.e., $\gamma_B^* = \bar{\gamma}_B$, then two scenarios may occur. Consider first the case where the unconstrained threshold in the full game is greater than the baseline model threshold (i.e., $\gamma_F^U > \delta\mathbb{E}[b]$). Recall that in the full game, the IC constrained is more relaxed and so the threshold in the full game $\gamma_F^* = \min\{\gamma_F^U, \bar{\gamma}_F\}$ must be greater than $\gamma_B^* = \bar{\gamma}_B$. The second case is when the unconstrained threshold in the full game is less than the baseline threshold (i.e., $\gamma_F^U < \delta\mathbb{E}[b]$). As the IC constraint is more relaxed in the full game, it must be that the unconstrained threshold in the full game is the actual threshold i.e., $\gamma_F^* = \gamma_F^U$ and is therefore less than the baseline threshold. ■

A.3 Extension: Public goods vs. bureaucratic output

In this section, we consider an extension of the model where the voter places different weights on public goods and bureaucratic output. Formally, let the voter's per-period payoff be $\kappa g_t + (1-\kappa)e_t$ where $\kappa \in (0,1)$. Note that the original setting correspond to $\kappa = \frac{1}{2}$.

Lemma 4. *$\bar{\gamma}_F, \bar{\gamma}_B, \gamma_B^U, \alpha^*$ are independent of κ, while γ_F^U is increasing in κ and converges monotonically to γ_F^U as $\kappa \to 1$.*

Proof of Lemma 4. κ does not factor into neither the politician's nor the bureaucrat's utilities. Therefore κ has no impact on $\bar{\gamma}_F, \bar{\gamma}_B$, or α^*, which depend only on the IC constraints for the politician and the bureaucrat. κ also does not impact γ_B^U since the voter's payoff in the benchmark depends only on g_t and her optimal choice is the same for any constant multiple of g_t. Now, in the full model, the voter's objective function takes the form

$$\max_c \left(1 - F(\gamma)\right)\left(\kappa\gamma + (1-\kappa)\alpha\lambda\right)$$

and γ_F^U solves the FOC:

$$\frac{1-F(\gamma)}{f(\gamma)} - \frac{1-\kappa}{\kappa}\alpha\lambda = \gamma$$

Since the LHS of the equation is decreasing in γ (and also increasing in κ) and RHS increasing in γ, it follows that the solution γ_F^U must be increasing in κ. Moreover, as $\kappa \to 1$, the FOC converges to

the FOC of the benchmark case and it follows γ_F^U converges to γ_B^U. ■

Proposition 5. *The optimal threshold in the benchmark game is independent of* κ *and the optimal threshold in the full game is increasing in* κ.

Proof of Proposition 5. Lemma 4 above implies that the optimal threshold in the benchmark game is independent of κ, since 1) the constraint on γ is independent of γ and 2) the unconstrained optimal threshold is independent of γ. On the other hand, in the full game the constraint is independent of κ but the unconstrained optimal is increasing in κ. This implies, given the strict quasi-concavity of the objective function, that the constrained optimal is increasing in κ as well. ■