Campaign Spending and Lobbying

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Abstract

We analyze a game-theoretic model of campaign spending and lobbying. Interest groups may spend money during the election to improve the electoral chances of candidates but may also spend money after the election to influence the policy that the winning candidate implements. Voters anticipate this lobbying and its effect on the final policy when choosing a candidate. When making campaign contributions today, interest groups must therefore anticipate how lobbying will affect the utility associated with electing each of the candidates from the perspectives of both voters and the interest groups themselves. We adopt extant technologies for each of the model's two stages in order to study the role that the polarization of interest groups and candidates have on expected campaign spending, lobbying expenditures, and final policy location in equilibrium. Further, we model several lobbying environments, each featuring different access to the politician. We find that policy moderation and campaign spending usually but not always move in opposite directions, both within lobbying regimes in response to exogenous changes but also when comparing across lobbying regimes. Our results also demonstrate that interest-group and candidate polarization must be considered jointly, as it is the relative values of these quantities that is most important for campaign spending and final policy location.

Word Count: 8304

1 Introduction

The Koch brothers' PAC Americans for Prosperity pledged to spend 400 million dollars on the 2018 U.S. midterm elections.[1] The campaign was designed to help their preferred candidates (mostly Republicans) win the elections. At the same, both brothers sit on the board of the American Legislative Executive Council (ALEC). ALEC writes model legislation for legislators to introduce, essentially a subsidy for adopting the preferred policy positions of the Kochs.

Seeking to affect policy through multiple channels is not unique to this one family. Tripathi, Ansolabehere, and Snyder (2002) and Lake (2015) show that the most influential interest groups routinely engage in both campaign spending and on lobbying. Further, much of the ire towards *Citizens United v. FEC* tacitly presumes, correctly, that interest groups are engaged in both campaign spending and lobbying.[2]

Indeed, there is no reason to assume these two instruments of influence are substitutes for one another or strategically independent. Campaign spending and lobbying affect policy in different but interrelated ways. Interest groups use campaign spending to help elect ideologically aligned politicians. After the election, though, the interest groups may lobby whichever politician won, increasing the affinity of an aligned candidate or moderating the policy of the less aligned candidate.

This paper analyzes the joint usage of these two alternative means by which to influence policy outcomes. It does so with a two-stage model of two interest groups, two candidates, and one representative voter. In the first stage, interest groups may spend on the campaign for either or both of the two candidates. In the second stage, interest groups may then lobby the winning candidate to affect the politician's policy choice.

We interpret campaign spending as improving the electoral chances of a candidate. This

[1] `https://thehill.com/homenews/campaign/371069-koch-network-to-spend-400-million-during-2018-midterm-`

[2] `https://www.nytimes.com/2010/01/22/us/politics/22donate.html`. The article also discusses the role of interest group extremism as increasingly important given the enhanced role interest groups would certainly be taking.

may be through advertising, Get Out The Vote (GOTV) initiatives, or other investments. The model captures this by supposing interest group expenditures before the election increase a candidate's valence. We assume lobbying takes the form of help drafting bills, effort with administrative tasks, or assistance justifying a stance to voters. Each interest group as well as the winning candidate may theoretically exert effort at this stage, though we consider several different lobbying environments. Through such efforts, lobbyists and politicians move policy closer to their own ideal points.

The campaign stage operates as an all-pay contest. Interest groups pay money to increase a candidate's valence. The candidate with the best (from the voter's perspective) combination of anticipated final policy and valence wins the election. In the second stage, lobbying takes the form of a "tug of war" as developed in Duggan and Gao (2018). The interest groups and winning politician exert effort to pull policy towards their respective ideal points; the final policy is a convex combination of these ideal points, weighted by each player's effort.

It is worth emphasizing that the politician is a strategic actor in the lobbying stage. An alternative approach might have the politician set forth a rule at the start of the game by which she weights interest group efforts relative to a fixed weighting and effort she attaches to her own ideal policy. Since we do not model a repeated setting, such a rule would not be credible. We thus find the assumption that the politician acts strategically under a fixed rule translating efforts into the final policy to be the more compelling assumption for the second stage. We discuss this further at points below.

Spending in the first stage is driven by difference in interest-group utility under the two candidates. The more interest groups have to gain by electing an aligned politician, we show, the higher is campaign spending. In equilibrium, voters and interest groups anticipate the effect of lobbying on final policy and the efforts to be exerted, and they vote and spend accordingly. The questions asked of the second stage is thus: how does the difference in final policies under the two candidates depend on the distribution of preferences among the

players, and how does the lobbying effort the groups exert depend on the spread of ideal points?

We first study campaign spending in the absence of any lobbying. The "no lobbying" context enables us to explore the mechanics of the campaign stage and also serves as a baseline against which to evaluate how campaign spending depends upon the presence of lobbying. The latter exercise is of interest from both modeling and policy perspectives.

Recognizing that politicians likely mete out access selectively, we begin to study lobbying by supposing an elected politician allows only ideologically-aligned interest groups to lobby her. We refer to this as "access lobbying." In this environment, relative ideological alignment and campaign contributions are each necessary and are together sufficient conditions for access to the politician.

Under access lobbying, interest groups that are more extreme than politicians intensify differences in politician ideal points, but if interest groups are relatively less polarized than politicians, lobbying exerts a moderating effect on the final policy. Indeed, while the polarization of politicians is important, it is not sufficient to determine equilibrium campaign spending or voter welfare. The ideological conflict between politicians and interest groups and between interest groups themselves also drives spending. The *relative polarization* between interest groups and politicians is more important than the level of polarization of politicians or interest groups when it comes to comparing the welfare implications of different lobbying environments.

If politicians become more polarized, whether they grow more or less aligned with their nearest interest group depends on whether or not the groups are relatively more extreme (i.e., polarized) than the politicians. Considering relative polarization helps us capture two important features of the ideological landscape that must both be taken into consideration to understand comparative statics. Specifically, campaign spending and policy moderation (in any given lobbying regime) depend on the degree of alignment between the left (right) group and left (right) politician, as well as the degree of disagreement between opposing

politicians and groups.

Next, we relax the requirements of alignment and contributions for groups to gain lobbying access to politicians. Specifically, we suppose that both interest groups have the same ability to influence the politician in the lobbying stage. We call this regime "open lobbying." Comparing the predictions under the two environments has implications for empirical work seeking to understand the role of ideological alignment for access to politicians.

Under open lobbying, opposing interest groups moderate one another. Despite an increase of lobbying activity, policy moderation is at its highest, and, as a result, campaign spending tends to be lower than in other lobbying environments. When an interest group not aligned ideologically with a politician is allowed to lobby her, it helps moderate the final policy. This is a boon for the median voter. In addition, because policy is more moderate regardless of who wins the election, the two potential final policies (under the different candidates) are more similar than with fewer lobbyists. This makes winning the election less important to the interest groups – even net of increased lobbying efforts – and drives down campaign spending. In fact, we uncover an imperfect but strong inverse correlation between policy moderation and campaign spending across regimes.

Finally, we analyze the effects of a limit on campaign contributions. We find that a strict limit does not affect policy at all. Interest groups spend less in the campaign stage, but act exactly the same in the lobbying stage. As long as the lobbying regime does not change, introducing limits on campaign contributions does not change lobbying behavior and therefore does not change policy.

2 Related Theoretical and Empirical Research

The closest model to ours is Felli and Merlo (2007). In their paper, interest groups can both support campaigns and lobby elected politicians. In contrast to this paper, however, interest groups never do both in equilibrium. Our paper shows how both campaign spending

and lobbying on behalf of the same politician can occur, exploring conditions under which they share or differ in their response to changes in exogenous factors. As stated above, the empirical literature consistently shows that interest groups spend money on both campaigns and lobbying for the same politicians (Tripathi, Ansolabehere, and Snyder 2002; Lake 2015), which is consistent with our model.

One way to conceptualize our model is with the inside/outside lobbying dynamic of Wolton 2018. Campaign spending (perhaps best thought of as advertising in this model) influences the public as *outside* lobbying. The lobbying in the second stage of our model, however, directly influences the politician and would thus be an example of *inside* lobbying.

Other models that use the all-pay contest framework for campaigns include Meirowitz (2008) and Ashworth and De Mesquita (2009). However, these papers let politicians themselves make campaign investments. We depart by modeling campaign contributions from interest groups, not politicians, as well as by adding a subsequent lobbying stage. Morton and Myerson (2012) do allow interest groups to contribute, but they also do not analyze a second, lobbying stage.

Our model differs from previous models that focused on informational campaign contributions and lobbying (Cotton 2012; Dahm and Porteiro 2008; Schnakenberg and Turner 2018; Bennedsen and Feldmann 2006). We specifically focus on policy-location motivated interest groups, and not information. Instead of only connecting lobbying to campaign spending through access, we allow both types of political spending to influence the final policy separately. This allows us to identify distinct but interrelated linkages between both campaign spending and lobbying and voter welfare.

Other papers model both contributions and lobbying, but purely within a contributions-as-access framework (Cotton 2009; Judd 2019). Our approach allows us to compare a variety of lobbying regimes and not focus solely on the direct contributions-as-access paradigm. Future work, however, ought to consider a combination of these approaches. In particular, adopting the approach discussed above in which a politician sets forth a rule that specifies

lobbyist influence in the second stage, the weight that interest group efforts in the second stage carry in determining the final policy (i.e., the access each group is granted) could depend upon contributions in the first stage.

This paper also contributes to the small but growing literature on interest group ideology and its policy implications. Bonica (2013) and Bonica (2014) develops a procedure to measure interest group ideology, while McKay (2010) and Thieme (2019) show that more extreme interest groups spend more on campaigns. These results are in line with this paper's results, especially in reference to the access lobbying regime.

The empirical literature has been mixed with regards to the effectiveness of political spending.[3] Ansolabehere, De Figueiredo, and Snyder Jr (2003), for example, advance the notion that contributions are not particularly effective, but may instead be a form of consumption. Our model helps rationalize the fact of relatively low amounts of observed money in politics (often known as Tullock's Paradox), even when money is very effective, due to a cancelling out effect under some lobbying regimes.

The rest of the paper proceeds as follows. First we describe the model and solve the baseline case with no lobbying. Next we analyze the access-lobbying environment and its implications for the campaign equilibrium. We then turn to the open-lobbying equilibrium and compare the three lobbying regimes. We conclude by discussing policy implications and related extensions, such as the effect of imposing of spending limits in the campaign stage.

3 Model Preliminaries

Three distinct groups of players interact within the model: interest groups (G), politicians/candidates (P), and voters. For simplicity, we consider a single representative voter, often referred to as the median voter or simply the voter (M). All players have an ideal point on the real line. No platform commitment is possible by politicians.

[3]While we focus mostly on the U.S. setting, the effects of political money are of interest world wide. Titl and Geys (2019), for example, show how donations to winning politicians increases a firm's chances of receiving procurement contracts in the Czech Republic.

The two candidates, one left-leaning candidate (P_L) and one right-leaning (P_R), have ideal points $\hat{x}_{P_R} = -\hat{x}_{P_L} = \pi > 0$. Similarly, there are two interest groups, one left-leaning (G_L) and one right-leaning (G_R), with ideal points $\hat{x}_{G_R} = -\hat{x}_{G_L} = \gamma > 0$. We set the voter's ideal point as $\hat{x}_M = 0$. The assumptions of symmetry in ideal points receive discussion below, alongside the interpretation of the parameters. We call P_j and G_i aligned if $i = j$.

The relative polarization/extremism of interest groups and politicians is of central interest. The only assumption on the relative magnitude of π and γ is that each aligned politician and group is closer ideologically than the groups are to one another. We define the concept of relative polarization and state the assumption explicitly. The assumption merely ensures that the interest groups would not ally together against the politicians to moderate policy beyond its final location in equilibrium. In fact, we only invoke the assumption when studying the open-lobbying environment.

Definition (Relative polarization)**.** *If $\gamma > (<)\pi$, the interest groups are more (less) polarized than the politicians.*

Assumption (Genuinely opposed interest groups)**.** *Let $\gamma > \pi/3$.*

The model has two distinct stages: an election stage and a policymaking stage. In the first stage, the interest groups simultaneously make their campaign spending decisions, where $s_{G_i,P_j} \geq 0$ is the contribution of interest group G_i to P_j. The voter then chooses a candidate. After the election, the winning politician P_j and both interest groups simultaneously make their policymaking/lobbying effort decisions, $e_{P_j}, e_{G_i}, i = L, R$. Finally, policy is implemented and outcomes are realized.

In addition to policy, the voter cares about politician valence. The total utility she receives if politician j wins the election with valence v_{P_j} and implements x_{P_j} is given by

$$U_M\left(x_{P_j}, v_{P_j}; \hat{x}_M\right) = -\left|x_{P_j} - \hat{x}_M\right| + v_{P_j} = -\left|x_{P_j}\right| + v_{P_j}.$$

Interest groups care about money spent on campaigns in addition to effort expended on

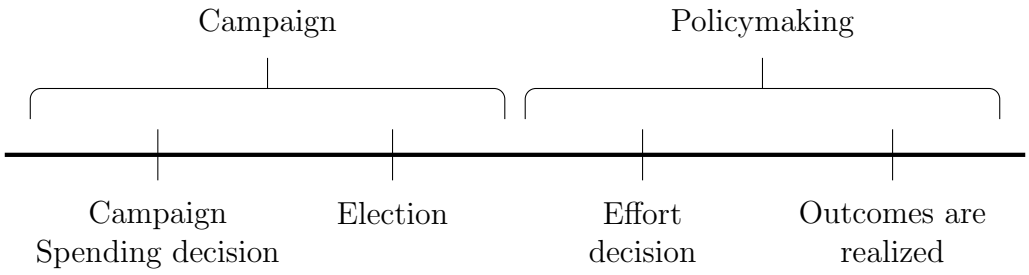

Figure 1: Sequence of Play

lobbying and the policy that is ultimately implemented. Campaign spending converts into valence linearly with P_j's valence being equal to:

$$v_{P_j} = \mu \cdot s_{P_j}, \tag{1}$$

where $\mu > 0$ and $s_{P_j} = \sum_{i=L,R} s_{G_i,P_j}$. The cost of lobbying enters quadratically into players utility functions. Group i's total utility from policy x_{P_j} being implemented, any campaign contributions it made, and the lobbying effort it exerted is given by:

$$U_{G_i}\left(x_{P_j}, s_{G_i,P_L}, s_{G_i,P_R}, e_{G_i}; c, \hat{x}_{G_i}\right) = -\left|x_{P_j} - \hat{x}_{G_i}\right| - s_{G_i,P_L} - s_{G_i,P_R} - c \cdot e_{G_i}^2.$$

Politicians only care about policy and effort put forth in the policymaking stage, such that politician j's utility from putting forth effort e_{P_j} and adopting final policy x_{P_j} is:

$$U_{P_j}\left(x_{P_j}, e_{P_j}; c, \hat{x}_{P_j}\right) = -\left|x_{P_j} - \hat{x}_{P_j}\right| - c \cdot e_{P_j}^2.$$

Lobbying takes the form of a tug of war game as presented in Duggan and Gao 2018 with (up to) three players. The winning politician and both interest groups make effort choices e_i. The final policy, denoted $\tilde{x}_{P_j}$, arises according to:

$$\tilde{x}_{P_j}(e_{G_L}, e_{G_R}, e_{P_j}; \hat{x}_{G_L}, \hat{x}_{G_R}, \hat{x}_{P_j}) = \frac{e_{P_j}\hat{x}_{P_j} + e_{G_L}\hat{x}_{G_L} + e_{G_R}\hat{x}_{G_R}}{e_{P_j} + e_{G_L} + e_{G_R}}. \tag{2}$$

A few comments are in order by way of interpretation. The multiplier on campaign spending, μ, measures the effectiveness of campaign spending; the higher μ is, the cheaper valence is to produce. The final policy is a convex combination of the winning candidate's and both interest groups' ideal points, weighted by the effort each put forward. Exerting more effort in the policymaking stage, then, pulls policy closer to one's ideal point (at a cost of $c \cdot e^2$).

The opportunity cost of money spent during the campaign stage or effort exerted in the lobbying stage includes all manner of other activities in which interest groups engage; for example, they may fund raise, conduct membership drives, or increase salaries. The translation of efforts in the policymaking supposes diminishing returns to lobbying. We treat the politician as a strategic player in this stage, treating the winning candidate as a lobbyist, essentially. Of course, the politician could enact whichever policy she wants, but we suppose that she must exert effort to marshal the resources necessary to counteract the interest groups and implement a policy closer to her ideal point than theirs. The tug-of-war technology is meant to capture just this. While we use the terms implement/enact, the politician need not be an executive. We may think of lobbyists vying to affect the policy position a legislator adopts.

We use the tug-of-war technology instead of a simple contracting scheme because of its applicability to more settings. The contest function allows us to analyze more than two players (i.e., one politician and one interest group) at the same time in a tractable way. The results with a linear contract scheme are qualitatively similar for the results about access, but the contracting model would not have also been amenable to analyzing the open lobbying regime, as well.

The reader will likely notice the strong symmetry assumptions around the ideal points of the candidates as well as the those of the interest groups. These effectively assume each pair (politicians, interest groups) are balanced ideologically around the median voter. These assumptions allow us to parsimoniously use relative polarization to discuss the polarization

of each pair as well as the extent to which a same-side politician and group are aligned.

With a setting of full information, the appropriate equilibrium concept is subgame perfect Nash equilibrium (hereafter just equilibrium). While this suggests the analysis proceed via backwards induction, it is instructive to first examine the campaign stage in the absence of subsequent lobbying, as it provides a baseline from which to compare campaigns with lobbying afterwards. Further, this approach highlights the most important features to focus on when studying the second stage, viz., the difference in utility for interest groups under their preferred and less-preferred candidates.

4 The Campaign Stage (the No-Lobbying Equilibrium)

We model the campaign stage as an all-pay contest, with the two interest groups simultaneously making valence investments. The voter then picks the candidate she prefers given each candidate's valence, ideal point, and the knowledge of what policy the candidate will implement in the second stage. In this section, we consider campaign spending absent lobbying in the second stage.

Remark 4.1. *In the absence of lobbying in the second stage, the winning politician implements her ideal point in the policymaking stage, i.e., the policy enacted will be* $\hat{x}_{P_R} = \pi$ *if the voter elects* P_R *and* $\hat{x}_{P_L} = -\pi$ *if the voter elects* P_L*.*

Interest groups spend on campaigns during the election and must pay regardless of whether their candidate wins. In line with intuition, then, our first result establishes that interest groups only contribute to aligned campaigns. This result serves us throughout the paper, as the premise that a candidate implements a policy closer to the aligned interest group's ideal point continues to hold when we introduce lobbying.[4]

[4]If, in addition to augmenting a candidate's valence and thus chance of winning, campaign contributions bought access to that candidate if elected, it may be the case that groups would contribute to both campaigns. While certainly interesting, this is beyond the scope of this analysis. We discuss this avenue for further research in our conclusion.

Lemma 4.1. *If the policy implemented by an aligned candidate will always be closer to an interest group's ideal point than the policy implemented by the unaligned candidate, then interest groups only contribute to the campaigns of aligned candidates.*

All proofs may be found in the appendix.

A few definitions will be helpful for the further analysis. We say an interest group is the *winner (loser)* if their ideologically-aligned candidate wins (loses) the election. Let $\overline{V}_{G_i}$ be the interest group's utility of winning the election and similarly let $\underline{V}_{G_i}$ be the interest group's utility of losing the election. Denote the difference $\overline{V}_{G_i} - \underline{V}_{G_i}$ with Δ_{G_i}, which we will often refer to as the value of winning for an interest group.

Because the politicians' ideal points are symmetric around 0, the two groups share the same value of winning. Indeed, this will continue to be true in subsequent sections because of the symmetry of the interest groups' ideal points around 0. As such, we will suppress the subscript for the winning value.

The symmetry of implemented policies around 0 also has implications for the voter's decision, as the next result makes clear. Because the voter is equidistant from both possible policies, she is indifferent between the two candidates from a policy perspective. Therefore she votes purely based on valence.

Lemma 4.2. *If the policies implemented by the two candidates are symmetric around the voter's ideal point, the candidate with the highest valence wins the election.*

No pure-strategy equilibria exist in this game. If, for example, the left interest group was going to spend Δ for sure, then the right interest group's best response is to spend nothing. If the right interest group was always going to spend 0, however, then the left interest group should clearly spend less than Δ and still win. Instead, the groups randomize over spending in equilibrium, and the distribution of spending has support from 0 up to the full benefit of winning the election. Such results are standard in the literature on (symmetric) all-pay auctions.

4.1 The No-Lobbying Equilibrium

Without lobbying, the value of winning the election is the based on the polarization of groups (politicians) if they are relatively less polarized than politicians (groups). In fact, when the interest groups are more extreme than the politicians, the value of winning is simply the difference between the politicians' ideal points. If the politicians are more extreme, however, the value of winning is instead the difference between the groups' ideal points.

Lemma 4.3. *With no lobbying in the second stage, the value of winning the election is given by:*

$$\Delta^N = \begin{cases} 2\gamma & \text{if } \gamma \leq \pi \\ 2\pi & \text{if } \gamma \geq \pi. \end{cases}$$

The starkness of this result stems primarily from the absolute-loss policy component of utility functions, but it nicely highlights the central logic animating campaign spending without lobbying. When interest groups are less polarized than politicians, some of the distance to their aligned candidate's ideal point cancels out the distance to the less-aligned candidate's ideal point, leaving interest group extremism as the only driver of the value of winning the election. When interest groups are more polarized than politicians, the distance to their more aligned candidate's ideal point is shared by the distance to the less-aligned candidate's ideal point, again washing out in the calculation of the value of winning the election.

Proposition 4.1. *With no lobbying in the second stage, the equilibrium spending of both interest groups is distributed uniformly over the interval* $[0, \Delta^N]$. *Expected spending for each interest group is* $\frac{\Delta^N}{2}$, *leading to expected total spending of* Δ^N.

While the multiplier μ does not appear in the expression for expected spending, we may easily observe that the expected total valence produced in equilibrium will be $\mu\Delta^N$. We discuss the interpretation of valence in greater depth below, in the section on voter welfare. To preface, however, we view valence as representing behavioral concerns. The role of these

non-policy considerations in the election increases in μ or whatever changes may lead to increases in Δ^N. The next result addresses the latter.

Corollary 4.1.1. *With no lobbying in the second stage, expected spending is increasing in politician polarization (π) when interest groups are more polarized than politicians. When interest groups are relatively less polarized than politicians, increasing politician polarization has no effect on expected spending.*

With no lobbying in the second stage, expected spending is increasing in interest group polarization (γ) when interest groups are less polarized than politicians. When interest groups are relatively more extreme than politicians, increasing group polarization has no effect on expected spending.

Changes in spending only result from local changes to $\min\{\gamma, \pi\}$; the negative effects of local changes to $\max\{\gamma, \pi\}$ on the utility of to a group of their aligned candidate winning the election are exactly outweighed by the positive effects on the utility to that group of their less aligned candidate winning. When interest groups are more extreme than politicians, the gain in proximity of the final policy to their own ideal points from electing their aligned candidate does not change as their own ideal points become more extreme.[5] It is fixed as the distance between the politicians' ideal points. When politicians are relatively more extreme, increasing politician polarization has two effects. It makes the aligned politician less appealing to the allied interest group; however, it also makes the misaligned politician less appealing to the same interest group. These two effects cancel out,[6] leaving expected spending at the same level.

Increasing the polarization of interest groups or politicians increases expected spending only if it increases the alignment of groups and politicians. Increasing polarization but decreasing alignment leads to no change in the value of winning. Turning to consider the lobbying stage, our analysis of the campaign stage highlights that the most relevant consid-

[5]The gain in utility would change if players were not risk neutral, but any such change is clearly not robust to the functional form of policy loss.

[6]See the previous footnote.

eration is how lobbying affects the value of winning for an interest group.

5 The Access-Lobbying Equilibrium

> "We had a hierarchy in my office in Congress. If you're a lobbyist who never gave us money, I didn't talk to you. If you're a lobbyist who gave us money, I might talk to you." – Mick Mulvaney[7]

In the lobbying stage, some subset of interest groups and the winning politician may expend effort to minimize the distance from the final policy – as given by Equation 2 – to their ideal points, taking into account the cost of effort, $c \cdot e^2$. Recall, we see such effort as representing more than just spending money. Instead, it also encompasses writing legislation, assigning staff, having meetings, justifying stances to constituents, and other effort-intensive legislative activities. Duggan and Gao (2018) develop and thoroughly explore this model of lobbying. Their results allow us to quickly establish several statements about the policy under each candidate, effort choices, and ultimately the value of winning for interest groups in equilibrium.

A plausible reading of the Mick Mulvaney quotation above is that contributions are a necessary but not sufficient condition for access to a politician, with ideological alignment being a likely additional necessary condition. In accordance with this view, we first explore an environment we refer to as "access lobbying." In this setting, only the interest group aligned with the winning candidate may participate (along with the candidate herself) in lobbying. McKay (2018b) and Kalla and Broockman (2016) present more systematic evidence that campaign contributions buy access, showing that politicians do not just implement policies favored by supported interest groups but also use language specifically written by groups that hosted fundraisers. Further, Fouirnaies and Hall (2018), Powell and Grimmer (2016), and McKay (2018a) show that interest group prioritize contributions for candidates that will

[7] https://www.vox.com/policy-and-politics/2018/4/25/17279244/mick-mulvaney-cfpb-lobbyist-donations-banks

hold positions on relevant committees. We subsequently consider "open lobbying," in which all interest groups may lobby the politician, regardless of ideological alignment or campaign contributions.

The lobbied policy in the access equilibrium is a convex combination of the winning politician and her aligned interest group's ideal points, weighted by their efforts. Specifically, in equilibrium the final policy splits the difference of these ideal points. We superscript key terms in this section with A.

Lemma 5.1. *If the right-leaning candidate wins the election, the final, equilibrium policy is* $\tilde{x}^A_{P_R} = \frac{\gamma+\pi}{2}$.

If instead the left-leaning candidate wins the election, the policy enacted in equilibrium is $\tilde{x}^A_{P_L} = -\frac{\gamma+\pi}{2}$.

Figure 2 shows that extreme interest groups pull the final policy away from median voter, as there is no counterweight from the losing interest group.

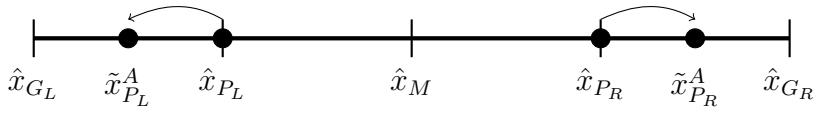

Figure 2: Lobbying Difference for Extreme Interest Groups

Moderate interest groups moderate an aligned but relatively extreme politician vis-à-vis the policy the politician would implement without any lobbying, as in Figure 3.

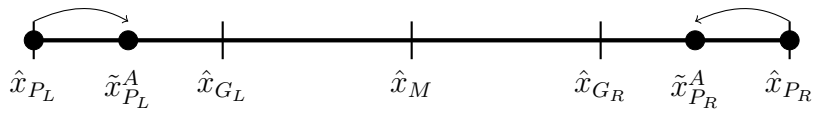

Figure 3: Lobbying Difference for Moderate Interest Groups

Proposition 5.1. *Under access lobbying, interest group equilibrium spending levels are distributed uniformly with support* $[0, \Delta^A]$ *such that expected total campaign spending is* Δ^A, *where*

$$\Delta^A = \begin{cases} \pi + \gamma - \frac{\gamma - \pi}{8} & if\ \pi \leq \gamma \\ 2\gamma - \frac{\pi - \gamma}{8} & if\ \pi \geq \gamma \end{cases}.$$

Note that the expression for Δ^A again depends on whether the interest groups are relatively moderate or extreme. When the interest groups are more polarized than politicians, the absolute polarization between the politicians plays almost as large a role in the value of winning the election as absolute polarization of the interest groups. However, if the politicians are relatively extreme instead, the interest group polarization accounts for the lion's share of the value of winning. The mechanism underlying this difference is as follows: when interest groups are less polarized than politicians, the anticipated final policies straddle each interest group's ideal point, leaving the distance between the interest groups as the primary determinant of the value of winning the election. The next result makes precise the way that equilibrium campaign spending depends on the polarization of interest groups and politicians.

Corollary 5.1.1. *Expected campaign spending under access lobbying increases in the polarization of interest groups, with the rate of increase slowing when interest groups are relatively more polarized than politicians.*

Expected campaign spending under access lobbying increases in the polarization of the candidates as long as politicians are less polarized than interest groups. When interest groups are relatively less polarized, spending falls in politician polarization

When politicians are relatively less polarized than interest groups, greater politician polarization has two effects. One, it makes policies more extreme, so there is a greater value to winning the election for the interest groups. Two, the politicians grow more aligned with interest groups and, therefore, equilibrium lobbying effort costs decrease, further increasing the value of winning.

Suppose now that politicians are relatively more polarized than interest groups and growing more so. Policies are again becoming more extreme. The decreased desirability of having

one's aligned candidate win the election is exactly offset by the increased disutility from having the opposing candidate win, such that there is no policy effect on the value of winning. Lobbying costs do increase as politicians and groups become less aligned, however, which exerts downward pressure on the value of winning. Expected campaign spending starts to drop.

We perform below a more full comparison of campaign spending under access lobbying and in the absence of lobbying. It is worth noting here, though, that strategic campaign spending must take into account subsequent lobbying. To consider campaign spending independently of lobbying risks mischaracterizing the effect of political ideology on both spending and policy.

6 The Open-Lobbying Equilibrium

We now consider open lobbying, in which we assume both interest groups lobby the winning candidate, regardless of her ideological alignment. While less realistic, the analysis of open lobbying serves two purposes. First, it may be more representative than access lobbying is of bygone, less-polarized eras. Second, and more importantly, the results herein demonstrate that a more contested lobbying environment need not imply higher campaign spending. Indeed, we find the opposite holds. The policy moderation induced under open lobbying leads to a sharp decrease in campaign spending. We employ the superscript O in this section.

Lemma 6.1. *Under open lobbying, the equilibrium policy if the voter elects the right-leaning candidate is*

$$\tilde{x}^O_{P_R} = \pi + 2\gamma - 2\sqrt{\gamma(\gamma+\pi)} \in (0, \min\{\gamma, \pi\}). \tag{3}$$

If the left-leaning candidate wins the election, the equilibrium policy is given by

$$\tilde{x}^O_{P_L} = 2\sqrt{\gamma(\gamma+\pi)} - 2\gamma - \pi \in (\max\{-\gamma, -\pi\}, 0). \tag{4}$$

Equilibrium policies lie in between the politicians' ideal points as well as in between the interest groups' ideal points. This is guaranteed because of the assumption of genuinely-opposed interest groups. The tug of war over the final policy results in policy moderation vis-à-vis politician or interest group ideal points. Figure 4 shows the anticipated final policies under the each of the potential candidates are now closer together as well as closer to the voter's ideal point. This significantly reduces the policy benefit of winning the election for the interest groups, though it remains positive. Because the losing group exerts more effort on policy than the winning group in equilibrium, the overall value of winning the election under open lobbying (Δ^O) is still ensured to be positive.

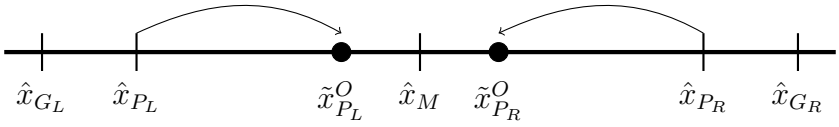

Figure 4: Location of policy after open lobbying

Note that the polarization of interest groups has a different effect than polarization of politicians (Figures 5 and 6). This is because interest groups participate in the lobbying stage regardless of whether their allied candidate wins the election. However, politicians only participate if they win the election.

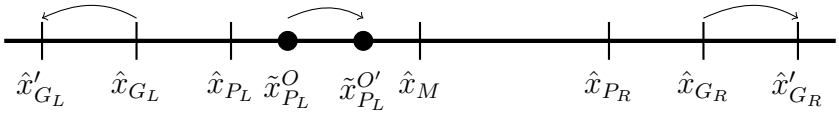

Figure 5: Change in open-lobbied policy location as γ increases

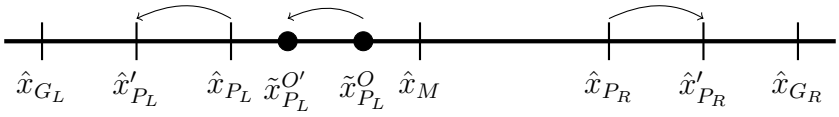

Figure 6: Change in open-lobbied policy location as π increases

Proposition 6.1. *Both groups' spending in the first stage is distributed uniformly over* $[0, \Delta^O]$. *Total expected campaign spending is thus* Δ^O, *where*

$$\Delta^O = (\sqrt{\pi+\gamma} - \sqrt{\gamma})^2 \left[2 + \frac{\sqrt{\gamma}}{\sqrt{\pi+\gamma}}\right]. \tag{5}$$

Corollary 6.1.1. *Under open lobbying, expected campaign spending is increasing in politician polarization, π, and decreasing in group polarization, γ.*

Opposing interest groups have the effect of moderating the final policy. The effort the opposing interest group exerts increases at a faster rate than the effort the aligned interest group exerts as the groups become more polarized. Therefore the more polarized the interest groups, the more moderate the open-lobbied policy will be, and the more similar the effort each group will need to exert in equilibrium regardless of the winning politician. As such, the value of winning the election falls in the extremism of interest groups, leading campaign spending in the first stage to fall.

No opposing politician moderates policy in the lobbying stage. Therefore, as politicians grow more extreme, the lobbied policies anticipated under the two candidates pull away from each other. Additionally, interest groups will exert less effort in equilibrium if their aligned candidate wins than if the opposing candidate wins. These two effects of increased politician polarization increase the value of winning for interest groups and thus lead to increased campaign spending in the first stage.

7 Policy and Empirical Implications

In this section we compare the three lobbying regimes considered above – open lobbying, access lobbying, and no lobbying – along two outcomes: campaign spending in the first stage and the moderation of policies implemented in the second stage. Figures 7, 8, 9, and 10 visually depict the above results. For each outcome, we consider changes in relative polarization first holding politician polarization fixed and then holding interest group polarization fixed. Each figure considers no lobbying (NL), access lobbying (AL), and open lobbying (OL).

The most policy-relevant question pertains to the comparison of access lobbying and no

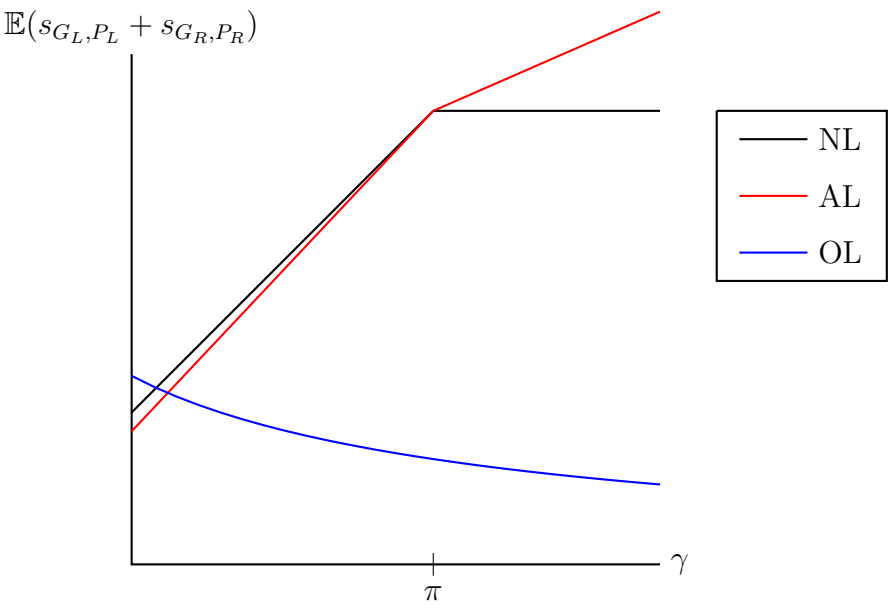

Figure 7: Expected Campaign Spending by Group Polarization

lobbying. Limits on lobbying are a more plausible counterfactual to the reality of access lobbying than open lobbying. It is hard to imagine a policy that required legislators to grant meaningful access to lobbyists or all stripes; it is easier to imagine a (call for a) reduction on lobbying activity.

While technically campaign spending (times the multiplier μ) and policy moderation both contribute positively to a voter's utility, we analyze these features separately. Policy moderation is clearly a boon for the voter. The use of valence in the model is less to capture actual voter well-being as much as a shorthand for behavioral or non-policy factors that play into voter decision making. On the whole, and in line with common narratives, we view campaign spending on valence (vis-à-vis the concept of informative campaign spending) as fundamentally inefficient. It is reasonable to suppose the voter benefits from greater policy moderation and less campaign spending. Regardless of the particular stance, however, analyzing these two outcomes separately allows us to generate insights about each as its own phenomenon of interest.

The first result below states that when interest groups are less polarized than politicians, no lobbying produces higher campaign spending and less moderate policies, with the opposite

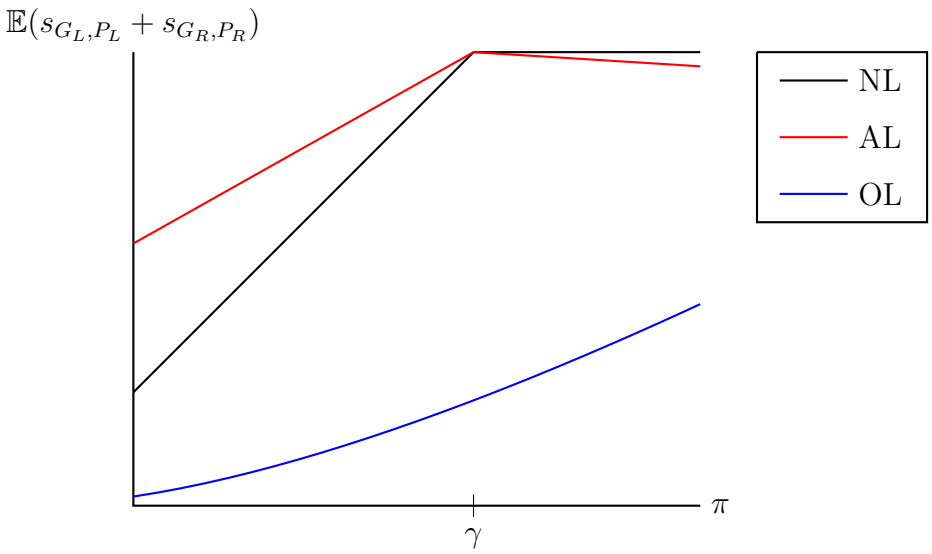

Figure 8: Expected Campaign Spending by Politician Polarization

being true when interest groups are more extreme than politicians. Two notable insights arise: first, there is not a trade-off between lower campaign spending and policy moderation – they go hand-in-hand, but, second, which institution achieves lower spending and greater policy moderation changes across the parameter space. If politicians are relatively more polarized than the relevant interest groups, access lobbying offers more moderation and reduces inefficient spending. If groups are relatively more extreme, however, a prohibition on lobbying would reduce spending and increase moderation.

Proposition 7.1. *If $\gamma < \pi$, policy moderation is greater and campaign spending is lower under access lobbying than if no lobbying occurs in the second stage.*

If $\gamma > \pi$, policy moderation is greater and campaign spending is lower if no lobbying occurs in the second stage than under access lobbying.

While access lobbying displays lower campaign spending than no lobbying when politicians are relatively more polarized than interest groups, it is relatively less of a reduction than no lobbying provides over access lobbying when groups are more polarized than politicians. It is only the increasing cost of lobbying effort that drives expenditures down as politicians grow more polarized than interest groups under access lobbying. Banning lobby-

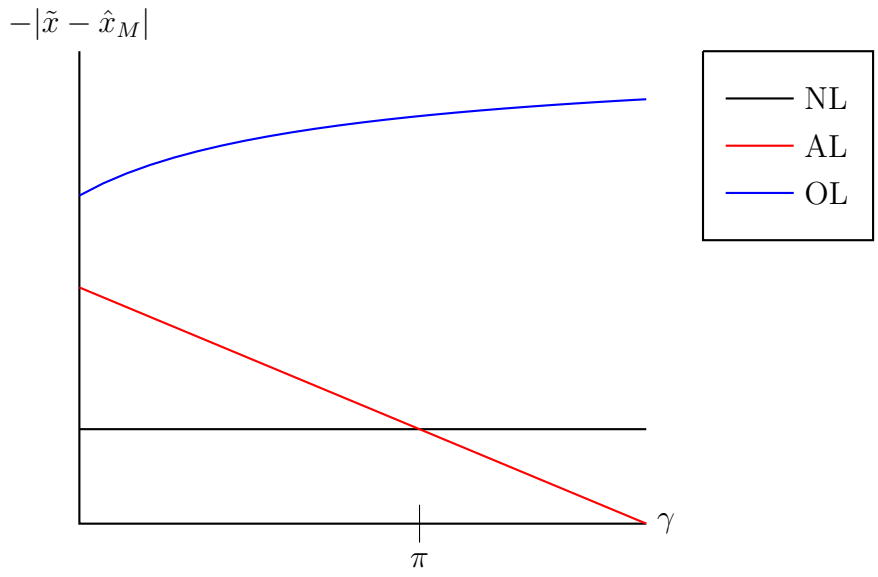

Figure 9: Voter Policy Utility by Group Polarization

ing would seem, on the whole, to lead to lower campaign spending than access lobbying. No such (weak) conclusions may be drawn with regards to policy moderation. Access lobbying is as preferable to no lobbying for policy moderation when groups are less extreme than politicians as no lobbying is preferable to access lobbying when groups are more polarized than politicians.

The second result indicates that open lobbying entails less campaign spending in equilibrium than no lobbying or access lobbying as long as politician polarization is less than or not too much larger than interest group polarization. Further, open lobbying always produces the most policy moderation of the three environments we consider. This further drives home that a given institution may induce both policy moderation and lower spending. Indeed, because policy moderation tends to reduce the value of winning, spending will often be lower as anticipated policies moderate. As with the result above, however, a single (viable) institution may not be the best choice for all values of relative polarization.

Proposition 7.2. *There exists a value of politician polarization $\overline{\pi}(\gamma) > \gamma$ such that if politicians are not more polarized this level, i.e., $\pi < \overline{\pi}(\gamma)$, campaign spending is lower under open lobbying than access lobbying or no lobbying environments.*

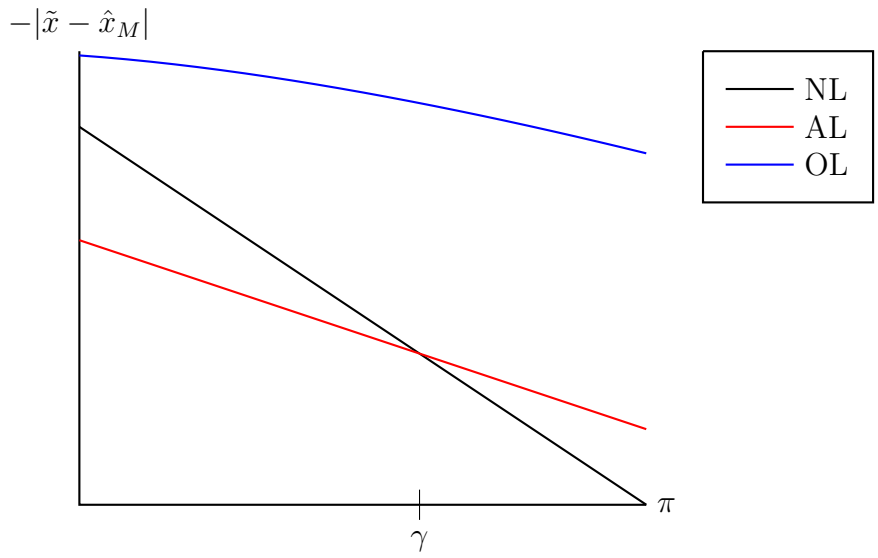

Figure 10: Voter Policy Utility by Politician Polarization

Policy moderation is always greater under open lobbying than under the access lobbying or no lobbying regimes.

Most interesting about open lobbying, perhaps, are the comparative statics that differ from those of access lobbying. These could be used shed light on whether politicians in fact limit access to ideologically aligned groups or whether they allow groups from across the spectrum to influence their policy positions. Specifically, policies grow less extreme as groups become more polarized under open lobbying but more extreme as groups become more polarized under access lobbying. Similarly, expected spending falls as groups grow more extreme under open lobbying but rises as groups grow more extreme under access lobbying. Certainly a more full consideration of open lobbying would consider the requirements for politicians to grant access to less aligned groups as well as aligned groups. Based on the results presented herein, however, evidence on spending discussed above suggests the lobbying environment in practice more closely resembles access lobbying than open lobbying.

7.1 Spending Caps

The preceding section shows how changes in expected spending correlate with changes in policy location. However, conditional on a specific candidate winning, the actual realized spending does not affect the final policy at all. This is true for all three lobbying regimes. Moreover, our predictions about spending pertain to expected spending – actual spending may be low or high.

As an example, consider access lobbying with extreme politicians (that is $\pi \geq \gamma$) and the right candidate winning the election. In this case, total expected spending is $2\gamma - \frac{\pi-\gamma}{8}$. Each candidate will spend something in the interval of $\left[0, 2\gamma - \frac{\pi-\gamma}{8}\right]$. Since the right candidate wins, the final policy will be $\frac{\gamma+\pi}{2}$.

However, the actual spending that gives this outcome can be anything as long as the right candidate spends more money. The left interest group could spend 0, and the right group couple spend γ. The left group could spend $\frac{\gamma}{2}$, and the right group could again spend γ. The left group could spend $2\gamma - \frac{\pi-\gamma}{16}$ while the right group spends $2\gamma - \frac{\pi-\gamma}{8}$.

These three cases have very different amounts of total spending. However, they all result in the exact same policy of $\frac{\gamma+\pi}{2}$. After determining the election winner, the final policy is the same regardless of the realized campaign spending. We can then take this logic further by analyzing strict campaign spending limits.

Proposition 7.3. *Let there be a limit κ on the amount interest groups can spend on the campaign. Then, if $\kappa \leq \Delta$, both interest groups always spend κ and win with probability $\frac{1}{2}$.*

When the spending limit is low enough, winning half of the time gives a positive expected utility (this is true for all three lobbying regimes). Therefore, both interest groups simply spend to the limit. However, the implemented policies are exactly the same as before the spending limits. The voter's policy utility hasn't changed at all even though there is now no association between campaign spending and policy outcomes. This serves as another demonstration that the money spent in the election may be a poor predictor of whether final

policy is moderate or extreme.

8 Conclusion

This paper demonstrates first and foremost that different means of political influence are not strategically independent decisions. Our results highlight how lobbying in the future affects the campaign contributions today. Specifically, interest group campaign contributions increase as the benefit of having an ideologically aligned politician in office rather than her opposition increases. Furthermore, polarization between candidates is not enough to explain campaign spending or equilibrium policy location. Candidate polarization relative to interest group polarization is a quantity as important to understanding campaign spending and lobbying effort as the absolute polarization of politicians or groups.

We considered the inter-dependency of campaign spending and lobbying under three different stylized lobbying regimes: 1) no lobbying, 2) access lobbying (in which only the aligned group may influence the politician's policy choice), and 3) open lobbying (in which aligned and unaligned interest groups have equal ability to influence the winning politician after the election). Our analysis suggests U.S. politics today may, in fact, suffer from too little lobbying, as we find open lobbying improves policy for the median voter and (nearly always) lowers campaign spending when compared to the other regimes. In this sense, one might wish to enable a wider spectrum of influence on politicians.

If facilitating a greater diversity of viewpoints to influence politicians is not feasible, then the optimal lobbying regime depends upon the relative polarization of interest groups and politicians. Banning lobbying may produce more or less extreme policies and lower or greater campaign spending than allowing access lobbying would. If groups are moderate relative to politicians, access lobbying moderates policies and lowers campaign spending. If instead politicians are relatively less extreme, no lobbying would obtain more moderate policies and lower expected campaign spending. As the latter situation may be more descriptive of the

present political system, our best option may be no lobbying if it is impossible to usher in an environment resembling open lobbying. We may think of no lobbying and open lobbying as being corner solutions, of sorts.

The comparative statics also have important implications for empirical research. First, we show how spending is conditional on the *relative* polarization between interest groups and politicians. Therefore studies that just look at politician ideology or just at interest group ideology may have an omitted variable problem. Researchers must consider interest group and politician ideology jointly to understand how they affect campaign spending.

Second, the lobbying effect on policy has a downstream effect on campaign spending. Therefore when considering the effect of campaign spending on policy, researchers must make they are not capturing the lobbying effect instead. These can be related, but distinct phenomenon. For example, just measuring spending against policy outcomes would miss that the final outcome is also a function of the lobbying regime.

Finally, spending should not be used a proxy for decreased voter welfare. Our results show that campaign spending and policy moderation often move in the same direction within and across lobbying regime. Further, depending on the realization of randomized spending, we may see much less or more spending than expected, all consistent with the same final policy, holding fixed the lobbying environment.

A number of potentially fruitful avenues for related work presented themselves throughout our analysis. Future work may wish to extend the model to speak to different limitations on spending, beyond the limit on campaign contributions analyzed above. Two additional such limitations come to mind. The first is a limit on lobbying expenditures, though presumably this would interpolate between the no lobbying and access lobbying regimes. The second would consist of a finite budget, such that interest groups face a potential trade-off between spending limited resources in the pre-election and post-election stages.

We did not allow politicians to make campaign investments in their own campaigns, instead focusing on interest group decision making. Certainly this comes at a loss of some

verisimilitude. How does interest group spending and lobbying change when politicians adopt a strategic role in the first stage? We also do not explicitly model the way in which campaign spending may purchase access. How might spending and the number of active lobbyists in the second stage change if contributions purchased a degree of access in addition to improving electoral chances of the receiving politician?

A final suggestion is to consider multiple political offices. Large interest groups give to multiple races at once. How does future lobbying of coalitions of politicians affect the distribution of spending across many simultaneous campaigns? The strategic interrelation of campaigns and lobbying points towards many avenues of interesting and worthwhile future research. This list comprises but a small sample of these.

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A Proofs

Proof of Lemma 4.1. Contributing to the unaligned candidate would reduce the effect of contributions to the aligned candidate while reducing utility in the second stage. ■

Proof of Lemma 4.2. This follows immediately from the voter's utility function. She incurs the same loss of utility from policies symmetric around her ideal point, $\hat{x}_M = 0$. Her best choice of candidate, then, is the one with the highest valence. ■

Proof of Lemma 4.3. If the interest groups are more polarized than the politicians, $\Delta^N = -(\gamma - \pi) - [-(\gamma - -\pi)] = \pi - \gamma + \gamma + \pi = 2\pi$.

If interest groups are relatively less polarized than politicans, then $\Delta^N = -(\pi - \gamma) - [-(\gamma - -\pi)] = \gamma - \pi + \gamma + \pi = 2\gamma$. ■

Proof of Proposition 4.1. Consider an affine transformation of interest group utility in which each is divided by Δ^N after having adding a constant such that interest group G_i (aligned with politician P_i) receives $\Delta^N - s_{G_i,P_i}$ if it wins the election and $-s_{G_i,P_i}$ if it loses. Substituting in $s_{G_i,P_i} = \frac{v_{P_i}}{\mu}$, we have utility functions that are equivalent to those in Equation (2) of Meirowitz (2008).[8] Proposition 2 from Meirowitz (2008) may then be applied, with $\frac{1}{\beta} = \mu\Delta^N$. To back out spending, however, we must divide equilibrium valence produced by $\frac{1}{\mu}$, such that spending is distributed uniformly over the interval $[0, \Delta^N]$.

It follows from the uniformly distributed spending that the expected campaign expenditures for each interest group are $\frac{\Delta^N}{2}$. With two candidates spending independently, total expected expenditures are simply Δ^N. ■

Proof of Corollary 4.1.1. If the interest groups are more polarized than the politicians, $\Delta^N = -(\gamma - \pi) - [-(\gamma - -\pi)] = \pi - \gamma + \gamma + \pi = 2\pi$.

If interest groups are relatively less polarized than politicans, then $\Delta^N = -(\pi - \gamma) - [-(\gamma - -\pi)] = \gamma - \pi + \gamma + \pi = 2\gamma$.

[8] Alternatively, one may reference nearly any introductory game theory textbook for these standard results on all-pay auctions.

The first derivative with respect to π is positive when interest groups are more polarized than politicians and 0 when groups are less polarized than politicians.

The first derivative with respect to γ is positive when politicians are more polarized than politicians and 0 when politicians are less polarized than groups. ■

Proof of Lemma 5.1. With two players who share the same per-unit cost of effort, the players choose symmetric effort levels and the tug-of-war produces a policy that splits the difference between their ideal points. ■

Proof of Proposition 5.1. When there are only two groups, we have the difference in final policies being $\pi + \gamma$, though the difference in utility must account for the lobbying cost incurred when granted access and also the possibility that the final policies are more extreme than the interest groups' ideal points. When interest groups are more extreme than politicians, the difference in policy utility is $\pi + \gamma$. When politicians are more extreme, (i.e., $\pi > \gamma$), the difference in policy utility is 2γ. The difference in policy utilities is increasing in γ even as the function changes.

The equilibrium effort is $e = \frac{|\gamma - \pi|}{2} / \sqrt{2c \cdot 2\frac{|\gamma - \pi|}{2}}$.

Equilibrium costs is therefore $c \cdot e^2 = \frac{|\gamma - \pi|}{8}$

The value of winning is thus is

$$\Delta^A = \begin{cases} 2\gamma - \frac{|\gamma - \pi|}{8} & \text{if } \pi > \gamma \\ \gamma + \pi - \frac{|\gamma - \pi|}{8} & \text{if } \pi < \gamma. \end{cases}$$

■

Proof of Corollary 5.1.1. The comparative statics follow from signing the derivatives of Δ^A with respect to γ and π, noting that increases in each variable holding fixed the other eventually toggles between the two cases. ■

Proof of Lemma 6.1. The equilibrium policies follow immediately from Duggan and Gao

(2018, p. 9). Specifically, the final policy equalizes the sum of squared distance of ideal points to the right of it and to the left of it. The final policy under the left politician, $\tilde{x}_{P_L}$, is such that

$$(\hat{x}_{P_L} - \tilde{x}_{P_L})^2 + (\hat{x}_{G_L} - \tilde{x}_{P_L})^2 = (\hat{x}_{G_R} - \tilde{x}_{P_L})^2.$$

The final policy under the right politician, $\tilde{x}_{P_R}$, is given according to

$$(\hat{x}_{G_L} - \tilde{x}_{P_R})^2 = (\hat{x}_{G_R} - \tilde{x}_{P_R})^2 + (\hat{x}_{P_R} - \tilde{x}_{P_R})^2.$$

Assumption 1 ensures that both groups' squared distance to the policy are not together set equal to a politician's squared distance to the policy, i.e., that $\tilde{x}^O_{P_R} \in \{0, \min\{\gamma, \pi\}\}$ and $\tilde{x}^O_{P_L} \in \{\max\{\gamma, \pi\}, 0\}$. ■

Proof of Proposition 6.1. The only aspect of the results that do not follow immediately from Lemmas 4.1 and 4.2 or adaptations of Proposition 4.1 and Corollary 4.1.1 are the value of and comparative statics on Δ^O. We set $A := (x_{G_R} - x_{G_L}) = 2\gamma$ and $B := (x_{P_R} - x_{G_L}) = \pi + \gamma = (x_{G_R} - x_{P_L}) =: C$, working in terms of A, B, C so as not to wash over effects with symmetry before necessary.

We first verify the open-lobbied policies are symmetric, i.e., the difference in voter utility from each of the potential, anticipated policies is zero:

$$\begin{aligned}
&- (x^*(P_R) + x^*(P_L)) = \\
&- (x_{P_R} + x_{P_L}) + \sqrt{2A}(\sqrt{B} - \sqrt{C}) = \\
&- (\pi - \pi) + \sqrt{2A}(\sqrt{B} - \sqrt{B}) = 0.
\end{aligned}$$

We then calculate the distance between the anticipated, open-lobbied policies, noting

that both will lie on the same side of a given interest group's ideal points.[9]

$$\begin{aligned}
x^*(P_R) - x^*(P_L) =& x_{P_R} + x_{G_R} - x_{G_L} - \sqrt{2}\sqrt{(x_{G_R} - x_{G_L})(x_{P_R} - x_{G_L})} \\
&-(\sqrt{2}\sqrt{(x_{G_R} - x_{G_L})(x_{G_R} - x_{P_L})} - x_{G_R} + x_{G_L} + x_{P_L}) \\
=& (x_{G_R} - x_{G_L}) + (x_{P_R} - x_{G_L}) + (x_{G_R} - x_{P_L}) \\
&- \left(\sqrt{2(x_{G_R} - x_{G_L})(x_{P_R} - x_{G_L})} + \sqrt{2(x_{G_R} - x_{G_L})(x_{G_R} - x_{P_L})}\right) \\
=& A + B + C - \sqrt{2AB} - \sqrt{2AC} \\
=& A + 2B - 2\sqrt{2AB} \\
=& (\sqrt{A} - \sqrt{2B})^2 \\
=& (\sqrt{2\gamma} - \sqrt{2(\pi + \gamma)})^2 \\
=& (\sqrt{2}(\sqrt{\pi + \gamma} - \sqrt{\gamma}))^2 \\
=& 2(\sqrt{\pi + \gamma} - \sqrt{\gamma})^2
\end{aligned}$$

The difference in costs for a group from electing an aligned candidate rather than the un-

[9] We need only do this calculation for one of the groups – the other is identical by symmetry.

aligned candidate is given by:

$$\begin{aligned}
c \cdot a^*_{G_R}(P_L)^2 - c \cdot a^*_{G_R}(P_R)^2 =& \frac{\left((x_{G_R} - x_{P_L}) + (x_{G_R} - x_{G_L}) - \sqrt{2}\sqrt{(x_{G_R} - x_{G_L})(x_{G_R} - x_{P_L})}\right)^2}{2\sqrt{2}\sqrt{(x_{G_R} - x_{G_L})(x_{G_R} - x_{P_L})}} \\
& - \frac{\left(\sqrt{2}\sqrt{(x_{G_R} - x_{G_L})(x_{P_R} - x_{G_L})} - (x_{P_R} - x_{G_L})\right)^2}{2\sqrt{2}\sqrt{(x_{G_R} - x_{G_L})(x_{P_R} - x_{G_L})}} \\
=& \frac{(A + C - \sqrt{2AC})^2}{2\sqrt{2AC}} - \frac{(\sqrt{2AB} - B)^2}{2\sqrt{2AB}} \\
=& \frac{A^2 + 2AB - 2A\sqrt{2AB}}{2\sqrt{2AB}} \\
=& \frac{(A - \sqrt{2AB})^2}{2\sqrt{2AB}} \\
=& \frac{A(\sqrt{A} - \sqrt{2B})^2}{2\sqrt{2AB}} \\
=& \frac{2\gamma(\sqrt{2\gamma} - \sqrt{2(\pi + \gamma)})^2}{2\sqrt{2(2\gamma(\pi + \gamma))}} = \frac{\sqrt{\gamma}}{\sqrt{\pi + \gamma}}(\sqrt{\pi + \gamma} - \sqrt{\gamma})^2
\end{aligned}$$

Finally, we put the policy and cost components together for the value of winning the election under open lobbying.

$$\begin{aligned}
\Delta^O = u_{G_R}(P_R) - u_{G_R}(P_L) =& (\sqrt{A} - \sqrt{2B})^2 + \frac{A(\sqrt{A} - \sqrt{2B})^2}{2\sqrt{2AB}} \\
=& (\sqrt{A} - \sqrt{2B})^2 \left[1 + \frac{A}{2\sqrt{2AB}}\right] \\
=& (\sqrt{2B} - \sqrt{A})^2 \left[1 + \frac{\sqrt{A}}{2\sqrt{2B}}\right] \\
=& 2(\sqrt{\pi + \gamma} - \sqrt{\gamma})^2 \left[1 + \frac{\sqrt{2\gamma}}{2\sqrt{\pi + \gamma}}\right] \\
=& (\sqrt{\pi + \gamma} - \sqrt{\gamma})^2 \left[2 + \frac{\sqrt{\gamma}}{\sqrt{\pi + \gamma}}\right]
\end{aligned}$$

■

Proof of 6.1.1. The comparative statics follow from signing the derivatives of Δ^O with re-

spect to γ and π.

$$\frac{\partial \Delta^O}{\partial \gamma} = -\frac{(\sqrt{\pi+\gamma}-\sqrt{\gamma})^2(\gamma+2\sqrt{\gamma}\sqrt{\pi+\gamma}+3(\gamma+\pi))}{2\sqrt{\gamma}\sqrt{\pi+\gamma}(\pi+\gamma)}$$

The two terms in the numerator are positive, as is the denominator, so the entire expression is negative.

$$\frac{\partial \Delta^O}{\partial \pi} = \frac{(\sqrt{\pi+\gamma}-\sqrt{\gamma})(\gamma+\sqrt{\gamma}\sqrt{\pi+\gamma}+4(\gamma+\pi))}{2\sqrt{\gamma}\sqrt{\pi+\gamma}(\pi+\gamma)}$$

Again, both expressions in the numerator are positive, but without a leading minus sign, the entire expression remains positive. ■

Proof of Proposition 7.1. If groups are less (more) polarized than politicians, then they moderate (make more extreme) policies under access lobbying.

If groups are less (more) polarized than politicians, then $\Delta^A < \Delta^O$ and vice versa. ■

Proof of Proposition 7.2. Spending under open lobbying is less than under no lobbying or access lobbying when $\gamma = \pi$.

$$\begin{aligned}
(\sqrt{2\gamma}-\sqrt{\gamma})^2\left[2+\frac{\sqrt{\gamma}}{\sqrt{2\gamma}}\right] = \\
\gamma(\sqrt{2}-1)^2\left(2+\frac{1}{\sqrt{2}}\right) < \\
\gamma(\sqrt{2}-1)\left(2+\frac{1}{\sqrt{2}}\right) < \\
\gamma(\sqrt{2}-1)(\sqrt{2}+1/2)\sqrt{2} < \\
\gamma(\sqrt{2}-1)(\sqrt{2}+1)\sqrt{2} = \\
\sqrt{2}\gamma < 2\gamma
\end{aligned}$$

From Corollaries 4.1.1, 5.1.1, and 6.1.1, when politicians are more polarized than interest groups, campaign spending is increasing under open lobbying but constant under no lobbying and decreasing under access lobbying, so there exists a level of politician polarization relative

to interest group polarization, $\overline{\pi}(\gamma)$, at which spending under open lobbying is greater than under the other two regimes. This is true when open lobbying spending is greater than no lobbying spending:

$$(\sqrt{\pi+\gamma}-\sqrt{\gamma})^2\left[2+\frac{\sqrt{\gamma}}{\sqrt{\pi+\gamma}}\right] \geq 2\gamma$$
$$\pi \geq \gamma\frac{1+\sqrt{5}}{2}$$

As long as the ratio of politician to interest group polarization is not larger than the golden ratio, then, open lobbying leads to the lowest expected campaign spending of the three regimes we consider.

Because the final policy is strictly closer to zero, the median voter's ideal point, than any of the three ideal points, it is clear that the open-lobbied policy is more moderate than either the politician's ideal point (under no lobbying) or the mid-point between the ideal points of an aligned politician and interest group (under access lobbying). ■

Proof of Proposition 7.3. Let κ be the maximum amount interest groups are allowed to spend on the campaign. Further, let Δ be the difference in utility between winning and losing for each interest group. Finally, assume $\kappa \leq \frac{\Delta}{2}$.

Consider a strategy profile where one groups spends κ and the other group spends $s_i < \kappa$. Then the losing group always loses and has an expected utility of less than $-s_i < 0$. They can then deviate to κ and get an expected utility of $\frac{\Delta}{2} - \kappa > 0$. A similar argument holds for deviations from κ to $s_i < \kappa$.

Now consider a mixed strategy profile where each interest group bids uniformly on the interval $[\underline{s}, \kappa]$. Each group wins with probability $\frac{1}{2}$ and has an expected utility of $\frac{\Delta}{2} - \frac{\kappa+\underline{s}}{2}$. However, an interest group can deviate to κ and win for sure. Then their expected utility is $\Delta - \kappa$ which is greater than $\frac{\Delta}{2} - \frac{\kappa+\underline{s}}{2}$.

Therefore both interest groups will play the pure strategy κ. ■