

Campaign Spending and Lobbying

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Abstract

We analyze a game-theoretic model of campaign spending and lobbying. Interest groups may spend money during the election to improve the electoral chances of candidates but may also spend money after the election to influence the policy that the winning candidate implements. Voters anticipate this lobbying and its effect on the final policy when choosing a candidate. When making campaign contributions today, interest groups must therefore anticipate how lobbying will affect the utility associated with electing each of the candidates from the perspectives of both voters and the interest groups themselves. We adopt extant technologies for each of the model's two stages in order to study the role that the polarization of interest groups and candidates have on expected campaign spending, lobbying expenditures, and final policy location in equilibrium. Further, we model several lobbying environments, each featuring different access to the politician. We find that policy moderation and campaign spending usually but not always move in opposite directions, both within lobbying regimes in response to exogenous changes but also when comparing across lobbying regimes. Our results also demonstrate that interest-group and candidate polarization must be considered jointly, as it is the relative values of these quantities that is most important for campaign spending and final policy location.

1 Introduction

The Koch brothers' PAC American for Prosperity pledged to spend 400 million dollars on the 2018 U.S. midterm elections.¹ The campaign was designed to help their preferred candidates (mostly Republicans) win the elections. At the same, both brothers sit on the board of the American Legislative Executive Council (ALEC). ALEC writes model legislation for legislators to introduce, essentially a subsidy for adopting the preferred policy positions of the Kochs.

Seeking to affect policy through multiple channels is not unique to this one family. Tripathi et al. (2002) and Lake (2015) show that the most influential interest groups routinely engage in both campaign spending and on lobbying. Further, much of the ire towards *Citizens United v. FEC* tacitly presumes, correctly, that interest groups are engaged in both campaign spending and lobbying.²

Indeed, there is no reason to assume these two instruments of influence are perfect substitutes for one another, or even substitutes at all. Campaign spending and lobbying affect policy in different but interrelated ways. Interest groups use campaign spending to help elect ideologically aligned politicians. After the election, though, the interest group may lobby whichever politician won, increasing the affinity of an aligned candidate or moderating the policy of the less aligned candidate.

This paper analyzes the joint usage of these two alternative means by which to influence policy outcomes. It does so with a two-stage model of two interest groups, two candidates, and one representative voter. In the first stage, interest groups may spend on the campaign for either or both of the two candidates. In the second stage, interest groups may then lobby the winning candidate to affect the politician's policy choice.

We interpret campaign spending as improving the electoral chances of a candidate. This

¹<https://thehill.com/homenews/campaign/371069-koch-network-to-spend-400-million-during-2018-midterm-election-cycle>

²<https://www.nytimes.com/2010/01/22/us/politics/22donate.html>. The article also discusses the role of interest group extremism as increasingly important given the enhanced role interest groups would certainly be taking.

may be through advertising, Get Out The Vote (GOTV) initiatives, or other investments. We capture this by supposing interest group expenditures before the election increase a candidate's valence. We assume lobbying takes the form of help drafting bills, effort with administrative tasks, or assistance justifying a stance to voters. Each interest group as well as the winning candidate may exert effort at this stage. Through such efforts, lobbyists and politicians move policy closer to their own ideal points.

The campaign stage operates as an all-pay contest, à la Meirowitz (2008), among others. Interest groups pay money to increase their favored candidate's valence. The candidate with the with the best (from the voter's perspective) combination of ideal point and valence wins the election. Lobbying then takes the form of a "tug of war," as in Duggan and Gao (2018). The interest groups and winning politician exert effort to pull policy towards their respective ideal points; the final policy is a convex combination of these ideal points, weighted by each player's effort.

Spending in the first stage is driven by difference in interest-group utility under the two candidates. The more interest groups have to gain by electing an aligned politician, the higher is campaign spending. The question asked of the second stage is thus: when is the difference in final policies greater/smaller for the interest groups, and how much lobbying effort do the groups exert? Voters and interest groups anticipate the effect of lobbying on final policy and vote/spend accordingly.

We first study campaign spending in the absence of any lobbying. The "no lobbying" context enables us to explore the mechanics of the campaign stage and also serves as a baseline against which to evaluate whether/how campaign spending changes in the presence of lobbying. The latter exercise is of interest from both modeling and policy perspectives.

Recognizing that politicians mete out access selectively, we begin to study lobbying by supposing an elected politician allows only ideologically-aligned interest groups to lobby her. We refer to this as "access lobbying." Relative ideological alignment and campaign contributions are each necessary and are together sufficient conditions for access to the

politician.

Under access lobbying, interest groups that are more extreme than politicians intensify differences in politician ideal points, but if interest groups are relatively less polarized than politicians, lobbying exerts a moderating effect on the final policy. Indeed, while the polarization of politicians is important, it is not sufficient to determine equilibrium campaign spending or voter welfare. The ideological conflict between politicians and interest groups and between interest groups themselves also drives spending. The *relative polarization* between interest groups and politicians is more important than the level of polarization of politicians or interest groups when it comes to comparing the welfare implications of different lobbying environments.

If politicians become more polarized, whether they grow more or less aligned with their nearest interest group depends on whether or not the groups are relatively more extreme (i.e., polarized) than the politicians. Considering relative polarization helps us capture two important features of the ideological landscape that must both be taken into consideration to understand comparative statics. Specifically, campaign spending and policy moderation (in any given lobbying regime) depend on the degree of alignment between the left (right) group and left (right) politician, as well as the degree of disagreement between opposing politicians and groups.

Finally, we relax the requirements of alignment and contributions for groups to gain lobbying access to politicians. Specifically, we suppose that both interest groups have the same ability to influence the politician in the lobbying stage. We call this regime “open lobbying.”

Under open lobbying, opposing interest groups moderate one another. Despite an increase of lobbying activity, policy moderation is at its highest, and as a result, campaign spending is (generally) at its lowest. The intuition is that when an interest group opposed to the politician is allowed to lobby, it helps moderate the final policy. This is a boon for the median voter. In addition, because policy is more moderate regardless of who wins the

election, the two potential final policies (under the different candidates) are more similar than with fewer lobbyists. This makes winning the election less important to the interest groups and drives down campaign spending. In fact, we uncover an imperfect but strong inverse correlation between policy moderation and campaign spending across regimes.

The closest model to ours is Felli and Merlo (2007). In their paper, interest groups can both support campaigns and lobby elected politicians. In contrast to this paper, however, interest groups never do both in equilibrium. Our paper shows how both campaign spending and lobbying on behalf of the same politician can occur, exploring conditions under which they share or differ in their response to changes in exogenous factors. As stated above, the empirical literature consistently shows that interest groups spend money on both campaigns and lobbying for the same politicians (Tripathi et al. (2002), Lake (2015)), which is consistent with our model.

One way to conceptualize our model is with the inside/outside lobbying dynamic of Wolton (2018). Campaign spending (perhaps best thought of as advertising in this model) influences the public as *outside* lobbying. Our version of lobbying, on the other hand, directly influences the politician and would be an example of *inside* lobbying.

Other models that use the all-pay contest framework for campaigns include Meirowitz (2008) and Ashworth and De Mesquita (2009). However, these papers let politicians themselves make campaign investments. We depart by letting third parties, not politicians contribute to campaigns. Morton and Myerson (2012) do allow interest groups to contribute, but they do not analyze a lobbying stage later.

Our model differs from previous models that focused on informational campaign contributions and lobbying (Cotton, 2012; Dahm and Porteiro, 2008; Schnakenberg and Turner, 2018; Bennedsen and Feldmann, 2006). We specifically focus on policy-location motivated interest groups, and not information. Instead of only connecting lobbying to campaign spending through access, we allow both types of political spending to influence the final policy separately. This allows us to identify a separate, indirect linkage between campaign

spending and lobbying and how it can affect voter welfare.

Other papers model both contributions and lobbying, but purely within a contributions-as-access framework (Cotton, 2009; Judd, 2019). Our approach allows us to compare a variety of lobbying regimes and not focus solely on the direct contributions-as-access paradigm.

This paper also contribute to the small but growing literature on interest group ideology and its policy implications. Bonica (2013, 2014) develop of procedure to measure interest group ideology, while McKay (2010) shows that more extreme interest groups spend more on campaigns. That result is in line with this paper’s access lobbying regime.

The empirical literature has been mixed with regards to the effectiveness of political spending.³ Ansolabehere et al. (2003), for example, advances the notion that contributions are not particularly effective, but may instead be a form of consumption. Our model helps rationalize the fact of relatively low amounts of observed money in politics (often known as Tullock’s Paradox), even when money is very effective due to a cancelling out effect under some lobbying regimes.

Finally, the use of “off-the-shelf” models – and especially simple, straightforward forms of these models – for each stage allows us to focus on understanding the role of interest group polarization relative to candidate polarization in interest groups’ decision over campaign spending and lobbying. We believe this approach draws productively on prior research and avoids proliferating new modeling technology unnecessarily. Moreover, for those interested in extensions of variations of the two-stage framework presented herein, the models in both stages are readily amenable to the incorporation of a variety of alternate assumptions and asymmetries.

The rest of the paper proceeds as follows. First we describe the model and solve the baseline case with no lobbying. Next we analyze the access-lobbying environment and its implications for the campaign equilibrium. We then turn to the open-lobbying equilibrium

³While we focus mostly on the U.S. setting, the effects of political money are of interest world wide. Titl and Geys (2019), for example, shows how donations to winning politicians increases a firm’s chances of receiving procurement contracts in the Czech Republic.

and compare the three lobbying regimes. We conclude by discussing policy implications and additional, related questions.

2 Model Preliminaries

Three distinct groups of players interact within the model: interest groups (G), politicians/candidates (P), and voters. For simplicity, we consider a single representative voter, often referred to as the median voter or simply the voter (M). All players have an ideal point on the real line. No platform commitment is possible by politicians.

The two candidates, one left-leaning candidate (P_L) and one right-leaning (P_R), have ideal points $\hat{x}_{P_R} = -\hat{x}_{P_L} = \pi > 0$. Similarly, there are two interest groups, one left-leaning (G_L) and one right-leaning (G_R), with ideal points $\hat{x}_{G_R} = -\hat{x}_{G_L} = \gamma > 0$. We set the voter's ideal point as $\hat{x}_M = 0$. The assumptions of symmetry in ideal points receive discussion below, alongside the interpretation of the parameters. We call P_j and G_i (relatively) aligned if $i = j$.

The relative polarization/extremism of interest groups and politicians is of central interest. The only assumption on the relative magnitude of π and γ is that each aligned politician and group is closer ideologically than the groups are to one another. We define the concept of relative polarization and state the assumption explicitly, which merely ensures that the interest groups would not ally together against the politicians to moderate policy beyond its final location in equilibrium.

Definition (Relative polarization). *If $\gamma > (<)\pi$, the interest groups are more (less) polarized than the politicians.*

Assumption (Genuinely opposed interest groups). *Let $\gamma > \pi/3$.*

The model has two distinct stages: an election stage and a policymaking stage. In the first stage, the interest groups simultaneously make their campaign spending decisions, where s_{G_i, P_j} is the contribution of interest group G_i to P_j . The voter then chooses a candidate. After

the election, the winning politician P_j and both interest groups simultaneously make their policymaking/lobbying effort decisions, $e_{P_j}, e_{G_i}, i = L, R$. Finally, policy is implemented and outcomes are realized.

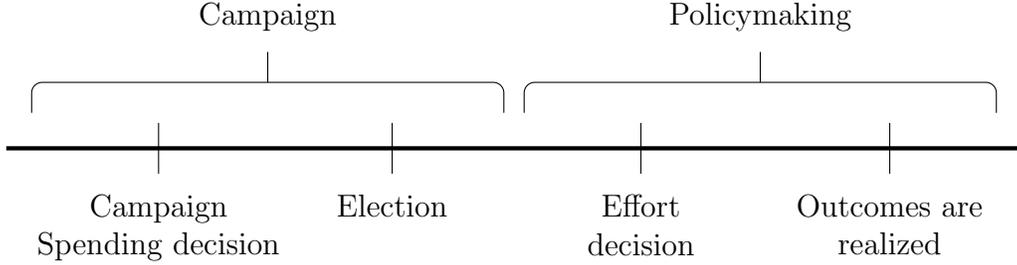


Figure 1: Game Timing

In addition to policy, the voter cares about politician valence. The total utility she receives if politician j wins the election with valence v_{P_j} and implements x_{P_j} is given by

$$U_M(x_{P_j}, v_{P_j}; \hat{x}_M) = -|x_{P_j} - \hat{x}_M| + v_{P_j} = -|x_{P_j}| + v_{P_j}.$$

Interest groups care about money spent on campaigns in addition to effort expended on lobbying and the policy that is ultimately implemented. Campaign spending converts into valence linearly with P_j 's valence being equal to:

$$v_{P_j} = \mu \cdot s_{P_j}, \tag{1}$$

where $\mu > 0$ and $s_{P_j} = \sum_{i=L,R} s_{G_i, P_j}$. The cost of lobbying enters quadratically into players utility functions. Group i 's total utility from policy x_{P_j} being implemented is thus:

$$U_{G_i}(x_{P_j}, s_{G_i, P_L}, s_{G_i, P_R}, e_{G_i}; c, \hat{x}_{G_i}) = -|x_{P_j} - \hat{x}_{G_i}| - s_{G_i, P_L} - s_{G_i, P_R} - c \cdot e_{G_i}^2.$$

Politicians only care about policy and effort put forth in the policymaking stage, such

that politician j 's utility from putting forth effort e_{P_j} and adopting final policy x_{P_j} is:

$$U_{P_j}(x_{P_j}, e_{P_j}; c, \hat{x}_{P_j}) = -|x_{P_j} - \hat{x}_{P_j}| - c \cdot e_{P_j}^2.$$

Lobbying takes the form of a tug of war game as presented in Duggan and Gao (2018) with three players. The winning politician and both interest groups make effort choices e_i . The final policy, denoted \tilde{x}_{P_j} , arises according to:

$$\tilde{x}_{P_j}(e_{G_L}, e_{G_R}, e_{P_j}; \hat{x}_{G_L}, \hat{x}_{G_R}, \hat{x}_{P_j}) = \frac{e_{P_j}\hat{x}_{P_j} + e_{G_L}\hat{x}_{G_L} + e_{G_R}\hat{x}_{G_R}}{e_{P_j} + e_{G_L} + e_{G_R}}. \quad (2)$$

A few comments are in order by way of interpretation. The multiplier on campaign spending, μ , measures the effectiveness of campaign spending; the higher μ is, the cheaper valence is to produce. The final policy is a convex combination of the winning candidate's and both interest groups' ideal points, weighted by the effort each put forward. Exerting more effort in the policymaking stage, then, pulls policy closer to one's ideal point (at a cost of $c \cdot e^2$).

The opportunity cost of money spent during the campaign stage or effort exerted in the lobbying stage includes all manner of other activities in which interest groups engage; for example, they may fund raise, conduct membership drives, or increase salaries. The translation of efforts in the policymaking supposes diminishing returns to lobbying. We treat the politician as a strategic player in this stage, treating the winning candidate as a lobbyist, essentially. Of course, the politician could enact whichever policy she wants, but we suppose that she must exert effort to marshal the resources necessary to counteract the interest groups and implement a policy closer to her ideal point than theirs. The tug-of-war technology is meant to capture just this. While we use the terms implement/enact, the politician need not be an executive. We may think of lobbyists vying to affect the policy position a legislator adopts.

The reader will likely notice the strong symmetry assumptions around the ideal points

of the candidates as well as the those of the interest groups, which effectively assume each pair (politicians, interest groups) are balanced ideologically. These assumptions allow us to parsimoniously use relative polarization to discuss the polarization of each pair as well as the extent to which a same-side politician and group are aligned. The biggest threat to generality is the symmetry the assumptions induce for the campaign stage. We show below that incorporating asymmetry changes none of the underlying logic of the results but does introduce a prohibitive arbitrariness for the comparative statics. As such, we maintain these assumptions throughout the discussion in-text.

With a setting of full information, the appropriate equilibrium concept is subgame perfect Nash equilibrium (hereafter just equilibrium). While this suggests the analysis proceed via backwards induction, it is instructive to first examine the campaign stage in the absence of subsequent lobbying, as it provides a baseline from which to compare campaigns with lobbying afterwards. Further, this approach highlights the most important features to focus on when studying the second stage, viz., the difference in utility for interest groups under their preferred and less-preferred candidates.

3 The Campaign Stage (the No-Lobbying Equilibrium)

We model the campaign stage as an all-pay contest, with the two interest groups simultaneously making valence investments. The voter then picks the candidate she prefers given each candidate's valence, ideal point, and the knowledge of what policy the candidate will implement in the second stage. In this section, we consider campaign spending absent lobbying in the second stage.

Remark 3.1. *In the absence of lobbying in the second stage, the winning politician implements her ideal point in the policymaking stage, i.e., the policy enacted will be $\hat{x}_{P_R} = \pi$ if the voter elects P_R and $\hat{x}_{P_L} = -\pi$ if the voter elects P_L .*

Interest groups spend on campaigns during the election and must pay regardless of

whether their candidate wins. In line with intuition, then, our first result establishes that interest groups only contribute to aligned campaigns. This result serves us throughout the paper, as the premise that a candidate implements a policy closer to the aligned interest group's ideal point continues to hold when we introduce lobbying.⁴

Lemma 3.1. *If the policy implemented by an aligned candidate will always be closer to an interest group's ideal point than the policy implemented by the unaligned candidate, then interest groups only contribute to the campaigns of aligned candidates.*

A few definitions will be helpful for the further analysis. We say an interest group is the *winner (loser)* if their ideologically-aligned candidate wins (loses) the election. Let \bar{V}_{G_i} be the interest group's utility of winning the election and similarly let \underline{V}_{G_i} be the interest group's utility of losing the election. Denote the difference $\bar{V}_{G_i} - \underline{V}_{G_i}$ with Δ_{G_i} , which we will often refer to as the value of winning for an interest group.

Because the politicians' ideal points are symmetric around 0, the two groups share the same value of winning. Indeed, this will continue to be true in subsequent sections because of the symmetry of the interest groups' ideal points around 0. As such, we will suppress the subscript for the winning value.

The symmetry of implemented policies around 0 also has implications for the voter's decision, as the next result makes clear. Because the voter is equidistant from both possible policies, she is indifferent between the two candidates from a policy perspective. Therefore she votes purely based on valence.

Lemma 3.2. *If the policies implemented by the two candidates are symmetric around the voter's ideal point, the candidate with the highest valence wins the election.*

No pure-strategy equilibria exist in this game. If, for example, the left interest group was going to spend Δ for sure, then the right interest group's best response is to spend nothing.

⁴If, in addition to augmenting a candidate's valence and thus chance of winning, campaign contributions bought access to that candidate if elected, it may be the case that groups would contribute to both campaigns. While certainly interesting, this is beyond the scope of this analysis. We discuss this avenue for further research in our conclusion.

If the right interest group was always going to spend 0, however, then the left interest group should clearly spend less than Δ and still win. Instead, the groups randomize over spending in equilibrium, and the distribution of spending has support from 0 up to the full benefit of winning the election.

3.1 The No-Lobbying Equilibrium

Without lobbying, the value of winning the election is based on the polarization of groups (politicians) if they are relatively less polarized than politicians (groups). In fact, when the interest groups are more extreme than the politicians, the value of winning is simply the difference between the politicians' ideal points. If the politicians are more extreme, however, the value of winning is instead the difference between the groups' ideal points.

Lemma 3.3. *With no lobbying in the second stage, the value of winning the election is given by:*

$$\Delta^N = \begin{cases} 2\gamma & \text{if } \gamma \leq \pi \\ 2\pi & \text{if } \gamma \geq \pi. \end{cases}$$

The starkness of this result stems primarily from the absolute-loss policy component of utility functions, but it nicely highlights the central logic animating campaign spending without lobbying. When interest groups are less polarized than politicians, some of the distance to their aligned candidate's ideal point cancels out the distance to the less-aligned candidate's ideal point, leaving interest group extremism as the only driver of the value of winning the election. When interest groups are more polarized than politicians, the distance to their more aligned candidate's ideal point is shared by the distance to the less-aligned candidate's ideal point, again washing out in the calculation of the value of winning the election.

Proposition 3.1. *With no lobbying in the second stage, the equilibrium spending of both interest groups is distributed uniformly over the interval $[0, \Delta^N]$, leading to expected total*

spending of Δ^N . Expected spending for each interest group is $\frac{\Delta^N}{2}$

While the multiplier μ does not appear in the expression for expected spending, we may easily observe that the expected total valence produced in equilibrium will be $\mu\Delta^N$. We discuss the interpretation of valence in greater depth below, in the section on voter welfare. To preface, however, we view valence as representing behavioral concerns. The role of these non-policy considerations in the election increases in μ or whatever changes may lead to increases in Δ^N . The next result addresses the latter.

Corollary 3.1.1. *With no lobbying in the second stage, expected spending is increasing in politician polarization (π) when interest groups are more polarized than politicians. When interest groups are relatively less polarized than politicians, increasing politician polarization has no effect on expected spending.*

With no lobbying in the second stage, expected spending is increasing in interest group polarization (γ) when interest groups are less polarized than politicians. When interest groups are relatively more extreme than politicians, increasing group polarization has no effect on expected spending.

Changes in spending only result from local changes to $\min\{\gamma, \pi\}$; the negative effects of local changes to $\max\{\gamma, \pi\}$ on the utility of to a group of their aligned candidate winning the election are exactly outweighed by the positive effects on the utility to that group of their less aligned candidate winning. When interest groups are more extreme than politicians, the gain in proximity of the final policy to their own ideal points from electing their aligned candidate does not change as their own ideal points become more extreme.⁵ It is fixed as the distance between the politicians' ideal points. When politicians are relatively more extreme, increasing politician polarization has two effects. It makes the aligned politician less appealing to the allied interest group; however, it also makes the misaligned politician less appealing to the same interest group. These two effects cancel out,⁶ leaving expected

⁵The gain in utility would change if players were not risk neutral, but any such change is clearly not robust to the functional form of policy loss.

⁶See the previous footnote.

spending at the same level.

Increasing the polarization of interest groups or politicians increases expected spending only if it increases the alignment of groups and politicians. Increasing polarization but decreasing alignment leads to no change in the value of winning. Turning to consider the lobbying stage, our analysis of the campaign stage highlights that the most relevant consideration is how lobbying affects the value of winning for an interest group.

4 The Access-Lobbying Equilibrium

“We had a hierarchy in my office in Congress. If you’re a lobbyist who never gave us money, I didn’t talk to you. If you’re a lobbyist who gave us money, I might talk to you.” – Mick Mulvaney⁷

In the lobbying stage, some subset of interest groups and the winning politician may expend effort to minimize the distance from the final policy – as given by Equation 2 – to their ideal points, taking into account the cost of effort, $c \cdot e^2$. Recall, we see such effort as representing more than just spending money. Instead, it also encompasses writing legislation, assigning staff, having meetings, selling stances to constituents, and other effort-intensive legislative activities. Duggan and Gao (2018) develop and thoroughly explore this model of lobbying. Their results allow us to quickly establish several statements about the policy under each candidate, effort choices, and ultimately the value of winning for interest groups in equilibrium.

A plausible reading of the Mick Mulvaney quotation above is that contributions are a necessary but not sufficient condition for access to a politician, with ideological alignment being a likely additional necessary condition. In accordance with this view, we first explore an environment we refer to as “access lobbying.” In this setting, only the interest group aligned with the winning candidate may participate (along with the candidate herself) in lobbying.

⁷<https://www.vox.com/policy-and-politics/2018/4/25/17279244/mick-mulvaney-cfpb-lobbyist-donations-banks>

McKay (2018b) and Kalla and Broockman (2016) present more systematic evidence that campaign contributions buy access, showing that politicians do not just implement policies favored by supported interest groups but also use language specifically written by groups that hosted fundraisers. Further, Fourinaies and Hall (2018), Powell and Grimmer (2016), and McKay (2018a) show that interest group prioritize contributions for candidates that will hold positions on relevant committees. We subsequently consider “open lobbying,” in which all interest groups may lobby the politician, regardless of ideological alignment or campaign contributions.

The lobbied policy in the access equilibrium is a convex combination of the winning politician and her aligned interest group’s ideal points, weighted by their efforts. Specifically, in equilibrium the final policy splits the difference of these ideal points. We superscript key terms in this section with A .

Lemma 4.1. *If the right-leaning candidate wins the election, the final, equilibrium policy is*

$$\tilde{x}_{P_R}^A = (\gamma + \pi)/2.$$

If instead the left-leaning candidate wins the election, the policy enacted in equilibrium is

$$\tilde{x}_{P_L}^A = -(\gamma + \pi)/2.$$

Figure 2 shows that extreme interest groups pull the final policy away from median voter, as there is no counterweight from the losing interest group.

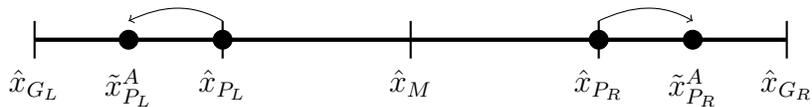


Figure 2: Lobbying Difference for Extreme Interest Groups

Moderate interest groups may also moderate an aligned but relatively extreme politician vis-à-vis the policy the politician would implement without any lobbying, as in Figure 3.

Proposition 4.1. *Under access lobbying, interest group equilibrium spending levels are distributed uniformly with support $[0, \Delta^A]$ such that expected total campaign spending is Δ^A , where*

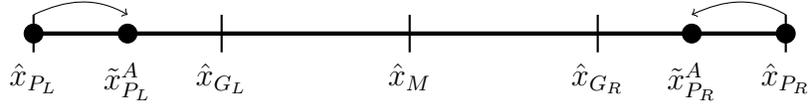


Figure 3: Lobbying Difference for Moderate Interest Groups

$$\Delta^A = \begin{cases} \pi + \gamma - \frac{\gamma - \pi}{8} & \text{if } \pi \leq \gamma \\ 2\gamma - \frac{\pi - \gamma}{8} & \text{if } \pi \geq \gamma \end{cases}$$

Note that the expression for Δ^A again depends on whether the interest groups are relatively moderate or extreme. When the interest groups are more polarized than politicians, the absolute polarization between the politicians plays almost as large a role in the value of winning the election as absolute polarization of the interest groups. However, if the politicians are relatively extreme instead, the interest group polarization accounts for the lion's share of the value of winning. The mechanism underlying this difference is as follows: when interest groups are less polarized than politicians, the anticipated final policies straddle each interest group's ideal point, leaving the distance between the interest groups as the primary determinant of the value of winning the election. The next result makes precise the way that equilibrium campaign spending depends on the polarization of interest groups and politicians.

Corollary 4.1.1. *Expected campaign spending under access lobbying increases in the polarization of interest groups, with the rate of increase slowing when interest groups are relatively more polarized than politicians.*

Expected campaign spending under access lobbying increases in the polarization of the candidates as long as politicians are less polarized than interest groups. When interest groups are relatively less polarized, spending falls in politician polarization

When politicians are relatively less polarized than interest groups, greater politician polarization has two effects. One, it makes policies more extreme, so there is a greater value to winning the election for the interest groups. Two, the politicians grow more aligned with

interest groups and, therefore, equilibrium lobbying effort costs decrease, further increasing the value of winning.

Suppose now that politicians are relatively more polarized than interest groups and growing more so. Policies are again becoming more extreme. The decreased desirability of having one's aligned candidate win the election is exactly offset by the increased disutility from having the opposing candidate win, such that there is no policy effect on the value of winning. Lobbying costs do increase as politicians and groups become less aligned, however, which exerts downward pressure on the value of winning. Expected campaign spending starts to drop.

We perform below a more full comparison of campaign spending under access lobbying and in the absence of lobbying. It is worth noting here, though, that strategic campaign spending must take into account subsequent lobbying. To consider campaign spending independently of lobbying risks mischaracterizing some comparative statics.

5 The Open-Lobbying Equilibrium

We now consider open lobbying, in which we assume both interest groups lobby the winning candidate, regardless of her ideological alignment. While less realistic, the analysis of open lobbying serves two purposes. First, it may be more representative than access lobbying is of bygone, less-polarized eras. Second, and more importantly, the results herein demonstrate that a more contested lobbying environment need not imply higher campaign spending. Indeed, we find the opposite holds. The policy moderation induced under open lobbying leads to a sharp decrease in campaign spending. We employ the superscript O in this section.

Lemma 5.1. *Under open lobbying, the equilibrium policy if the voter elects the right-leaning candidate is*

$$\tilde{x}_{PR}^O = \pi + 2\gamma - 2\sqrt{\gamma(\gamma + \pi)} \in (0, \min\{\gamma, \pi\}). \quad (3)$$

If the left-leaning candidate wins the election, the equilibrium policy is given by

$$\tilde{x}_{P_L}^O = 2\sqrt{\gamma(\gamma + \pi)} - 2\gamma - \pi \in (\max\{-\gamma, -\pi\}, 0). \quad (4)$$

Equilibrium policies lie in between the politicians' ideal points as well as in between the interest groups' ideal points. This is guaranteed because of the assumption of genuinely-opposed interest groups. The tug of war over the final policy results in policy moderation vis-à-vis politician or interest group ideal points. Figure 4 shows the anticipated final policies under the each of the potential candidates are now closer together as well as closer to the voter's ideal point. This significantly reduces the policy benefit of winning the election for the interest groups, though it remains positive. Because the losing group exerts more effort on policy than the winning group in equilibrium, the overall value of winning the election under open lobbying (Δ^O) is still ensured to be positive.

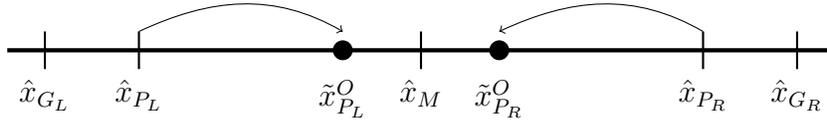


Figure 4: Location of policy after open lobbying

Note that the polarization of interest groups has a different effect than polarization of politicians (Figures 5 and 5). This is because interest groups participate in the lobbying stage regardless of whether their allied candidate wins the election. However, politicians only participate if they win the election.

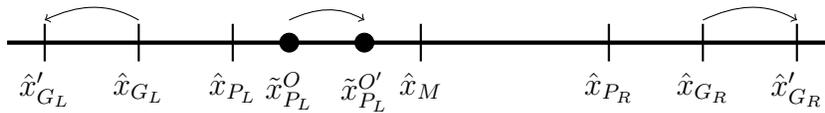


Figure 5: Change in γ

Proposition 5.1. *Both groups' spending in the first stage is distributed uniformly over $[0, \Delta^O]$. Total expected campaign spending is thus Δ^O , where*

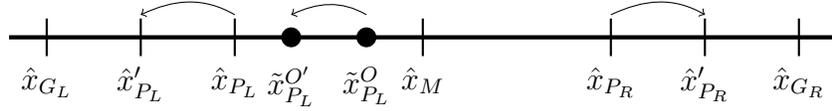


Figure 6: Change in π

$$\Delta^O = (\sqrt{\pi + \gamma} - \sqrt{\gamma})^2 \left[2 + \frac{\sqrt{\gamma}}{\sqrt{\pi + \gamma}} \right]. \quad (5)$$

Corollary 5.1.1. *Under open lobbying, expected campaign spending is increasing in politician polarization, π , and decreasing in group polarization, γ .*

Opposing interest groups have the effect of moderating the final policy. The effort the opposing interest group exerts increases at a faster rate than the effort the aligned interest group exerts as the groups become more polarized. Therefore the more polarized the interest groups, the more moderate the open-lobbied policy will be, and the more similar the effort each group will need to exert in equilibrium regardless of the winning politician. As such, the value of winning the election falls in the extremism of interest groups, leading campaign spending in the first stage to fall.

No opposing politician moderates policy in the lobbying stage. Therefore, as politicians grow more extreme, the lobbied policies anticipated under the two candidates pull away from each other. Additionally, interest groups will exert less effort in equilibrium if their aligned candidate wins. These two effects of increased politician polarization increase the value of winning for interest groups and thus lead to increased campaign spending in the first stage.

6 Comparisons

In this section we compare the three lobbying regimes considered above – open lobbying, access lobbying, and no lobbying – along two outcomes: campaign spending in the first stage and the moderation of policies implemented in the second stage.

The most policy-relevant question pertains to the comparison of access lobbying and no

lobbying. Limits on lobbying are a more plausible counterfactual to the reality of access lobbying than open lobbying. It is hard to imagine a policy that required legislators to grant meaningful access to lobbyists or all stripes; it is easier to imagine a (call for a) reduction on lobbying activity.

While technically campaign spending (times the multiplier μ) and policy moderation both contribute positively to a voter's utility, we analyze these features separately. Policy moderation is clearly a boon for the voter. The use of valence in the model is less to capture actual voter well-being as much as a shorthand for behavioral or non-policy factors that play into voter decision making. On the whole, and in line with common narratives, we view campaign spending on valence (vis-à-vis the concept of informative campaign spending) as fundamentally inefficient. The voter thus likely benefits from greater policy moderation and less campaign spending. Regardless of the particular stance, however, analyzing these two outcomes separately allows us to generate insights about each as its own phenomenon of interest.

The first result below states that when interest groups are less polarized than politicians, no lobbying produces higher campaign spending and less moderate policies, with the opposite being true when interest groups are more extreme than politicians. Two notable insights arise: first, there is not a trade-off between lower campaign spending and policy moderation – they go hand-in-hand, but, second, which institution achieves lower spending and greater policy moderation changes across the parameter space. If politicians are relatively more polarized than the relevant interest groups, access lobbying offers more moderation and reduces inefficient spending. If groups are relatively more extreme, however, a prohibition on lobbying would reduce spending and increase moderation.

Proposition 6.1. *If $\gamma < \pi$, policy moderation is greater and campaign spending is lower under access lobbying than if no lobbying occurs in the second stage.*

If $\gamma > \pi$, policy moderation is greater and campaign spending is lower if no lobbying occurs in the second stage than under access lobbying.

The second result indicates that open lobbying entails less campaign spending in equilibrium than no lobbying or access lobbying as long as politician polarization is less than or not too much larger than interest group polarization. Further, open lobbying always produces the most policy moderation of the three environments we consider. This further drives home that a given institution may induce both policy moderation and lower spending. Indeed, because policy moderation tends to reduce the value of winning, spending will often be lower as anticipated policies moderate. As with the result above, however, a single (viable) institution may not be the best choice for all values of relative polarization.

Proposition 6.2. *There exists a value of politician polarization $\bar{\pi}(\gamma) > \gamma$ such that if politicians are not more polarized this level, i.e., $\pi < \bar{\pi}(\gamma)$, campaign spending is lower under open lobbying than access lobbying or no lobbying environments.*

Policy moderation is always greater under open lobbying than under the access lobbying or no lobbying regimes.

Figures 7, 8, 9, and 10 visually depict the above results. For each outcome, we consider changes in relative polarization first holding politician polarization fixed and then holding interest group polarization fixed. Each figure considers no lobbying, access lobbying, and open lobbying.

Proposition 6.3. *The voter always prefers policies under open lobbying to those under no lobbying and access lobbying. No lobbying produces more moderate policies than access lobbying when interest groups are extreme relative to politicians. When politicians are more polarized than interest groups, however, access lobbying produces policies closer to the voter's ideal point than no lobbying.*

Realized Spending and Policy

The preceding section shows how changes in expected spending correlate with changes in policy location. However, conditional on a specific candidate winning, the actual realized

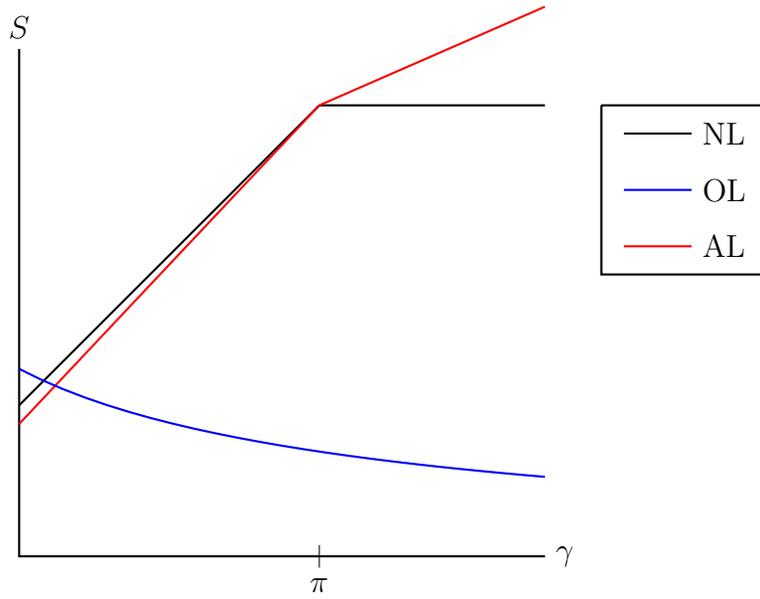


Figure 7: Expected Campaign Spending by Group Polarization

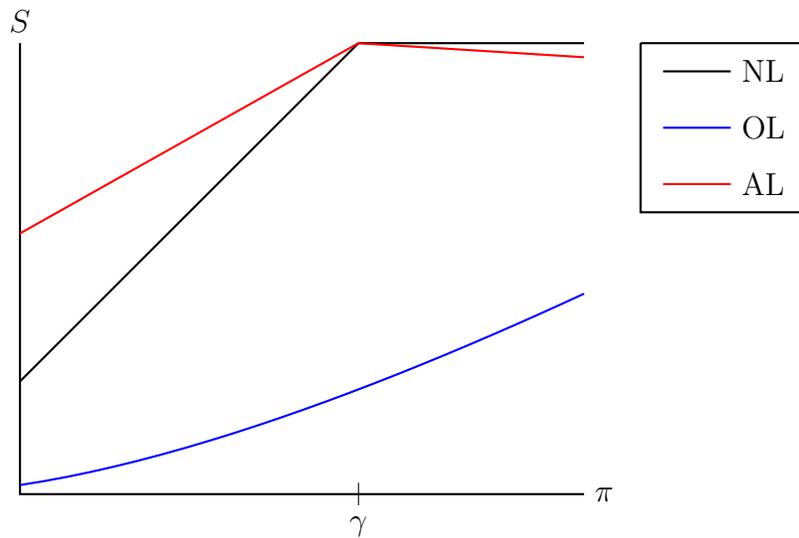


Figure 8: Expected Campaign Spending by Politician Polarization

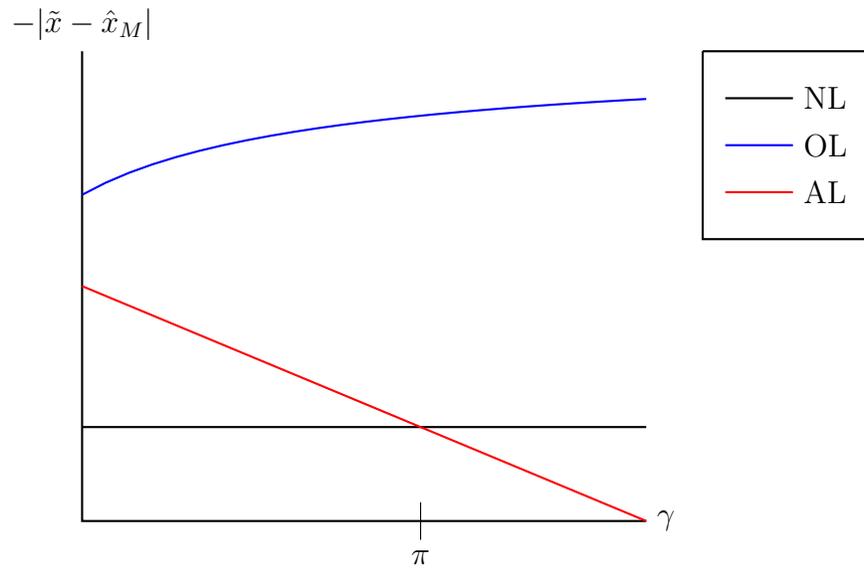


Figure 9: Voter Policy Utility by Group Polarization

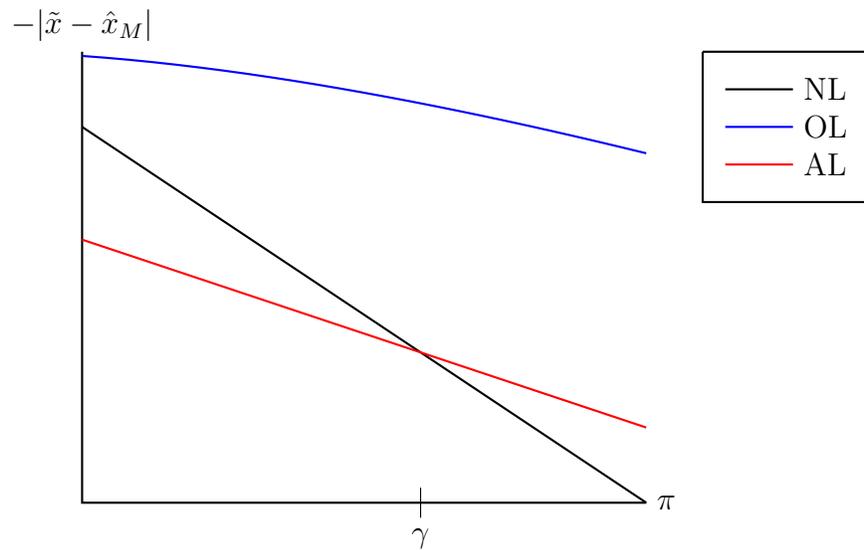


Figure 10: Voter Policy Utility by Politician Polarization

spending does not affect the final policy at all. This is true for all three lobbying regimes. Moreover, our predictions about spending pertain to expected spending – actual spending may be low or high.

As an example, consider access lobbying with extreme politicians (that is $\pi \geq \gamma$) and the right candidate winning the election. In this case, total expected spending is $2\gamma - \frac{\pi-\gamma}{8}$. Each candidate will spend something in the interval of $[0, 2\gamma - \frac{\pi-\gamma}{8}]$. Since the right candidate wins, the final policy will be $\frac{\gamma+\pi}{2}$.

However, the actual spending that gives this outcome can be anything as long as the right candidate spends more money. The left interest group could spend 0, and the right group couple spend γ . The left group could spend $\frac{\gamma}{2}$, and the right group could again spend γ . The left group could spend $2\gamma - \frac{\pi-\gamma}{16}$ while the right group spends $2\gamma - \frac{\pi-\gamma}{8}$.

These three cases have very different amounts of total spending. However, they all result in the exact same policy of $\frac{\gamma+\pi}{2}$. After determining the election winner, the final policy is the same regardless of the realized campaign spending. This serves as yet another demonstration that the money spent in the election may not be the best predictor of where final policy is actually located.

7 Conclusion

This paper demonstrates first and foremost that different means of political influence are not independent decisions, and they ought to be studied in tandem. Our results highlight how lobbying in the future affects the campaign contributions today. Specifically, interest group campaign contributions increase as the benefit of having an ideologically aligned politician in office rather than her opposition increases. Furthermore, polarization between candidates is not enough to explain campaign spending or equilibrium policy location. The interaction of candidate polarization and interest group polarization drives both campaign spending and lobbying effort.

We consider the inter-dependency of campaign spending and lobbying under three different stylized lobbying regimes: 1) no lobbying, 2) access lobbying (in which only the aligned group may influence the politician's policy choice), and 3) open lobbying (in which aligned and unaligned interest groups have equal ability to influence the winning politician after the election). One insight from this comparative analysis is that U.S. politics today may, in fact, suffer from too little lobbying. Moving from access lobbying to open lobbying improves policy for the median voter and (nearly always) lowers campaign spending. In this sense, one might wish to enable a wider spectrum of influence on politicians.

In particular, more formal roles for lobbyists should be considered. By bringing in interest groups from across the ideological spectrum in an official capacity, lawmakers may be able to incorporate ideas from all sides. This may lead to more moderate policies. By not carving out an official space for interest groups, we may not be able to move away from the access lobbying regime.

If facilitating a greater diversity of viewpoints to influence politicians is not feasible, then the optimal lobbying regime depends upon the relative polarization of interest groups and politicians. If groups are moderate relative to politicians, access lobbying moderates policies and lowers campaign spending. If instead politicians are relatively less extreme, no lobbying would obtain more moderate policies and lower expected campaign spending. As the latter situation may be more descriptive of the present political system, our best option may be no lobbying if it is impossible to usher in an environment resembling open lobbying. We may think of no lobbying and open lobbying as being corner solutions, of sorts.

The comparative statics have important implications for empirical research. First, we show how spending is conditional on the *relative* polarization between interest groups and politicians. Therefore studies that just look at politician ideology or just at interest group ideology may have an omitted variable problem. Researchers must consider interest group and politician ideology jointly to understand how they affect campaign spending.

Second, the lobbying effect on policy has a downstream effect on campaign spending.

Therefore when considering the effect of campaign spending on policy, researchers must make they are not capturing the lobbying effect instead. These can be related, but distinct phenomenon. For example, just measuring spending against policy outcomes would miss that the final outcome is also a function of the lobbying regime.

Finally, spending should not be used a proxy for decreased voter welfare. Our results show that campaign spending and policy moderation often move in the same direction within and across lobbying regime. Further, depending on the realization of randomized spending, we may see much less or more spending than expected, all consistent with the same final policy holding fixed the lobbying environment.

A number of potentially fruitful avenues for related work presented themselves throughout our analysis. Because we built productively upon extant models, each of which feature several extensions/variations already worked out, extensions/variations motivated by cases not covered by our model are likely achievable. What of interest groups that are much more moderate than politicians (more moderate than our assumptions allowed)? How do they allocate resources to campaigns expenditures and lobbying efforts?

While certain asymmetries might be worth considering, an online appendix presents results for asymmetric candidate and group locations.⁸ Our sense from these exercises is that the complexity of these cases is not prohibitive, but the comparative statics for expected spending depend on arbitrary (rather than substantive) assumptions about the location of each player. We suspect that the most fruitful insights maintain the symmetry assumptions but consider wider parameter spaces and, especially, different preferences and additional institutional arrangements.

Future work may also wish to extend the model to speak to different limitations on spending. Three such limitations come to mind. The first is a finite budget, such that interest groups face a potential tradeoff between spending limited resources in the pre-election and post-election stages. The second is a limit on campaign contributions, most interestingly

⁸Forthcoming.

compared to the third, an analogous regime that instead limits lobbying expenditures.

We do not allow politicians to make campaign investments in themselves, instead focusing on interest group decision making. Certainly this comes at a loss of some verisimilitude. How does interest group spending and lobbying change when politicians adopt a strategic role in the first stage?

We also do not explicitly model the way in which campaign spending may purchase access. How might spending and the number of active lobbyists in the second stage change if contributions purchased a degree of access in addition to improving electoral chances of the receiving politician.

A final suggestion is to consider multiple political offices. Large interest groups give to multiple races at once. How does future lobbying of coalitions of politicians affect the distribution of spending across many simultaneous campaigns? The linking of campaigns and lobbying points towards many avenues of interesting and worthwhile future research.

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A Proofs

Proof of Lemma 3.1. Contributing to the unaligned candidate would reduce the effect of contributions to the aligned candidate while reducing utility in the second stage. ■

Proof of Lemma 3.2. This follows immediately from the voter's utility function. She incurs the same loss of utility from policies symmetric around her ideal point, $\hat{x}_M = 0$. Her best choice of candidate, then, is the one with the highest valence. ■

Proof of Lemma 3.3. If the interest groups are more polarized than the politicians, $\Delta^N = -(\gamma - \pi) - [-(\gamma - -\pi)] = \pi - \gamma + \gamma + \pi = 2\pi$.

If interest groups are relatively less polarized than politicians, then $\Delta^N = -(\pi - \gamma) - [-(\gamma - -\pi)] = \gamma - \pi + \gamma + \pi = 2\gamma$. ■

Proof of Proposition 3.1. Consider an affine transformation of interest group utility in which each is divided by Δ^N after having adding a constant such that interest group G_i (aligned with politician P_i) receives $\Delta^N - s_{G_i, P_i}$ if it wins the election and $-s_{G_i, P_i}$ if it loses. Substituting in $s_{G_i, P_i} = v_{P_i}/\mu$, we have utility functions that are equivalent to those in Equation (2) of Meirowitz (2008). Proposition 2 from Meirowitz (2008) may then be applied, with $\frac{1}{\beta} = \mu\Delta^N$. To back out spending, however, we must divide equilibrium valence produced by $1/\mu$, such that spending is distributed uniformly over the interval $[0, \Delta^N]$.

It follows from the uniformly distributed spending that the expected campaign expenditures for each interest group are $\frac{\Delta^N}{2}$. With two candidates spending independently, total expected expenditures are Δ^N . ■

Proof of Corollary 3.1.1. If the interest groups are more polarized than the politicians, $\Delta^N = -(\gamma - \pi) - [-(\gamma - -\pi)] = \pi - \gamma + \gamma + \pi = 2\pi$.

If interest groups are relatively less polarized than politicians, then $\Delta^N = -(\pi - \gamma) - [-(\gamma - -\pi)] = \gamma - \pi + \gamma + \pi = 2\gamma$.

The first derivative with respect to π is positive when interest groups are more polarized than politicians and 0 when groups are less polarized than politicians.

The first derivative with respect to γ is positive when politicians are more polarized than politicians and 0 when politicians are less polarized than groups. ■

Proof of Lemma 4.1. With two players who share the same per-unit cost of effort, the players choose symmetric effort levels and the tug-of-war produces a policy that splits the difference between their ideal points. ■

Proof of Proposition 4.1. When there are only two groups, we have the difference in final policies being $\pi + \gamma$, though the difference in utility must account for the lobbying cost incurred when granted access and also the possibility that the final policies are more extreme than the interest groups' ideal points. When interest groups are more extreme than politicians, the difference in policy utility is $\pi + \gamma$. When politicians are more extreme, (i.e., $\pi > \gamma$), the difference in policy utility is 2γ . The difference in policy utilities is increasing in γ even as the function changes.

The equilibrium effort is $e = \frac{\gamma - \pi}{2} / \sqrt{2c \cdot 2^{\frac{\gamma - \pi}{2}}}$.

Equilibrium costs is therefore $c \cdot e^2 = \frac{|\gamma - \pi|}{8}$

The value of winning is thus is

$$\Delta^A = \begin{cases} 2\gamma - \frac{|\gamma - \pi|}{8} & \text{if } \pi > \gamma \\ \gamma + \pi - \frac{|\gamma - \pi|}{8} & \text{if } \pi < \gamma. \end{cases}$$

■

Proof of Corollary 4.1.1. The comparative statics follow from signing the derivatives of Δ^A with respect to γ and π , noting that increases in each variable holding fixed the other eventually toggles between the two cases. ■

Proof of Lemma 5.1. The policies follow immediately from Duggan and Gao (2018, p. 9). ■

Proof of Proposition 5.1. The only aspect of the results that do not follow immediately from Lemmas 3.1 and 3.2 or adaptations of Proposition 3.1 and Corollary 3.1.1 are the value of and comparative statics on Δ^O . We set $A := (x_{G_R} - x_{G_L}) = 2\gamma$, $B := (x_{P_R} - x_{G_L}) = \pi + \gamma = (x_{G_R} - x_{P_L}) =: C$, and $D := (x_{P_R} - x_{P_L}) = 2\pi$, working in terms of A, B, C so as not to wash over effects with symmetry before necessary.

We first verify the open-lobbied policies are symmetric, i.e., the difference in voter utility from each of the potential, anticipated policies is zero:

$$\begin{aligned}
& - (x^*(P_R) + x^*(P_L)) = \\
& - (x_{P_R} + x_{P_L}) + \sqrt{2A}(\sqrt{B} - \sqrt{C}) = \\
& - (\pi - \pi) + \sqrt{2A}(\sqrt{B} - \sqrt{B}) = 0.
\end{aligned}$$

We then calculate the distance between the anticipated, open-lobbied policies, noting that both will lie on the same side of a given interest group's ideal points.⁹

$$\begin{aligned}
x^*(P_R) - x^*(P_L) &= x_{P_R} + x_{G_R} - x_{G_L} - \sqrt{2}\sqrt{(x_{G_R} - x_{G_L})(x_{P_R} - x_{G_L})} \\
& - (\sqrt{2}\sqrt{(x_{G_R} - x_{G_L})(x_{G_R} - x_{P_L})} - x_{G_R} + x_{G_L} + x_{P_L}) \\
&= (x_{G_R} - x_{G_L}) + (x_{P_R} - x_{G_L}) + (x_{G_R} - x_{P_L}) \\
& - \left(\sqrt{2(x_{G_R} - x_{G_L})(x_{P_R} - x_{G_L})} + \sqrt{2(x_{G_R} - x_{G_L})(x_{G_R} - x_{P_L})} \right) \\
&= A + B + C - \sqrt{2AB} - \sqrt{2AC} \\
&= A + 2B - 2\sqrt{2AB} \\
&= (\sqrt{A} - \sqrt{2B})^2 \\
&= (\sqrt{2\gamma} - \sqrt{2(\pi + \gamma)})^2 \\
&= (\sqrt{2}(\sqrt{\pi + \gamma} - \sqrt{\gamma}))^2 \\
&= 2(\sqrt{\pi + \gamma} - \sqrt{\gamma})^2
\end{aligned}$$

⁹We need only do this calculation for one of the groups – the other is identical by symmetry.

The difference in costs for a group from electing an aligned candidate rather than the unaligned candidate is given by:

$$\begin{aligned}
c \cdot a_{G_R}^*(P_L)^2 - c \cdot a_{G_R}^*(P_R)^2 &= \frac{\left((x_{G_R} - x_{P_L}) + (x_{G_R} - x_{G_L}) - \sqrt{2} \sqrt{(x_{G_R} - x_{G_L})(x_{G_R} - x_{P_L})} \right)^2}{2\sqrt{2} \sqrt{(x_{G_R} - x_{G_L})(x_{G_R} - x_{P_L})}} \\
&\quad - \frac{\left(\sqrt{2} \sqrt{(x_{G_R} - x_{G_L})(x_{P_R} - x_{G_L})} - (x_{P_R} - x_{G_L}) \right)^2}{2\sqrt{2} \sqrt{(x_{G_R} - x_{G_L})(x_{P_R} - x_{G_L})}} \\
&= \frac{(A + C - \sqrt{2AC})^2}{2\sqrt{2AC}} - \frac{(\sqrt{2AB} - B)^2}{2\sqrt{2AB}} \\
&= \frac{A^2 + 2AB - 2A\sqrt{2AB}}{2\sqrt{2AB}} \\
&= \frac{(A - \sqrt{2AB})^2}{2\sqrt{2AB}} \\
&= \frac{A(\sqrt{A} - \sqrt{2B})^2}{2\sqrt{2AB}} \\
&= \frac{2\gamma(\sqrt{2\gamma} - \sqrt{2(\pi + \gamma)})^2}{2\sqrt{2}(2\gamma(\pi + \gamma))} = \frac{\sqrt{\gamma}}{\sqrt{\pi + \gamma}} (\sqrt{\pi + \gamma} - \sqrt{\gamma})^2
\end{aligned}$$

Finally, we put the policy and cost components together for the value of winning the election under open lobbying.

$$\begin{aligned}
\Delta^O = u_{G_R}(P_R) - u_{G_R}(P_L) &= (\sqrt{A} - \sqrt{2B})^2 + \frac{A(\sqrt{A} - \sqrt{2B})^2}{2\sqrt{2AB}} \\
&= (\sqrt{A} - \sqrt{2B})^2 \left[1 + \frac{A}{2\sqrt{2AB}} \right] \\
&= (\sqrt{2B} - \sqrt{A})^2 \left[1 + \frac{\sqrt{A}}{2\sqrt{2B}} \right] \\
&= 2(\sqrt{\pi + \gamma} - \sqrt{\gamma})^2 \left[1 + \frac{\sqrt{2\gamma}}{2\sqrt{\pi + \gamma}} \right] \\
&= (\sqrt{\pi + \gamma} - \sqrt{\gamma})^2 \left[2 + \frac{\sqrt{\gamma}}{\sqrt{\pi + \gamma}} \right]
\end{aligned}$$

■

Proof of 5.1.1. The comparative statics follow straightforwardly from signing the derivatives of Δ^O with respect to γ and π . [See attached Mathematica notebook.] ■

Proof of Proposition 6.2. Spending under open lobbying is less than under no lobbying or access lobbying when $\gamma = \pi$. (See attached Mathematica notebook.)

From Corollaries 3.1.1, 5.1.1, and 4.1.1, when politicians are more polarized than interest groups, campaign spending is increasing under open lobbying but constant under no lobbying and decreasing under access lobbying, so there exists a politician polarization relative to interest group polarization, $\bar{\pi}(\gamma)$, at which spending under open lobbying is greater than under the other two regimes. As seen in Figure 8, $\bar{\pi}(\gamma)$ may very well be less than 3γ , as required by the assumption that $\gamma < \pi/3$. ■

Proof of Proposition 6.3. The open lobbying policies are always in between both interest group ideal points as well as in between both politician ideal points.

If the politicians are less polarized than the interest groups, no lobbying produces more moderate ideal points than access lobbying, which would pull the policy away from the median voter. The opposite is true when politicians are more polarized than interest groups, as access lobbying pulls the policies closer to the voter's ideal point than no lobbying. ■