Delegation to an Overconfident Expert

Scott Ashworth[*] Greg Sasso[†]

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Abstract

Policymakers often delegate partial decision-making authority to experts. Although monetary transfers can align an expert's policy choices with the decision maker's preferences, such transfers are typically not observed in practice. We analyze delegation in a principal-agent model, allowing transfers. The policymaker and expert have identical preferences over state-contingent policy, but disagree over the accuracy of the expert's information. Specifically, the policymaker believes the expert is overconfident in the precision of the signal he receives about the state of the world. The optimal mechanism is a delegation interval, and transfers are not used.

[*]Harris School of Public Policy Studies, University of Chicago, email: sashwort@uchicago.edu

[†]Bocconi University, email: gregory.sasso@unibocconi.it

Policy-making inevitably involves delegating some decision-making power to better informed experts. Strikingly, explicit monetary incentives are rarely used in political delegation. This creates a puzzle for standard incentive theory. Indeed, Baron (2000) and Krishna and Morgan (2008) show that monetary transfers can efficiently ameliorate the conflict created by the different preferences of the policymaker and expert in the canonical Crawford and Sobel (1982) model.

We suggest a new rationale for avoiding monetary transfers in delegation. We model a policymaker and an expert who agree that the expert knows more about the effects of policies, but disagree about how much more.[1] The policymaker and the expert do not disagree about goals—they have the same underlying preferences over outcomes. We characterize the optimal mechanism, assuming monetary transfers are possible, subject to the expert's limited liability.

We focus on overprecision—the expert has greater confidence in her information's accuracy than the policymaker thinks warranted. Studies in psychology show, across many domains, that people (including experts) are more confident in their probabilistic judgments than their information warrants (Griffin and Tversky, 1992).[2] For example, Haran, Moore and Morewedge (2010) interpret the lack of overlap in the predictive confidence intervals of climate experts (reported by Morgan and Keith (1995)) as evidence of overprecision about the possible ranges of both temperature outcomes and the environmental impacts of climate change. Based on such evidence, Keller and Nicholas (2013, p. 12) argue that "[t]he resulting overconfidence can lead to risk estimates that are biased toward smaller values and, as a result, too-small investments in risk management".

Our main result shows that monetary transfers are not used at all when the policymaker believes the expert suffers from overprecision. A delegation window, in which the expert chooses policy from an interval of allowed choices, is optimal.[3] The policymaker (partially) controls the expert by choosing an interval that excludes the more extreme choices the expert might make.

Our result contrasts sharply with those of Baron (2000) and Krishna and Morgan (2008). They study the standard additive-bias model. In that model, the expert observes the state perfectly,

[1]Hirsch (2016) also models disagreement between a principal and agent. In his model, the disagreement is over policy fundamentals, rather than over information about policy fundamentals.

[2]Ortoleva and Snowberg (2015) study a model of overprecision and voter behavior.

[3]Our result compliments other findings that a delegation window is optimal even when transfers are possible. Gailmard (2009) argues that a delegation window will be optimal when the legislature cannot make an ex-ante commitment to monetary incentives. Our result shows that, in a substantively salient case, such a commitment, even if feasible, is not valuable. (Diermeier and Feddersen (1998) and Callander (2008) study other issues related to the possibility of commitment in a legislative context.) Our analysis is also related to the economics literature on contracting with overconfidence (Sandroni and Squintani, 2007; Grubb, 2009; De la Rosa, 2011).

and conflict arises from the different preferences of the policymaker and the expert. In our model, the expert observes a noisy signal of the state, and conflict arises from disagreement about just how noisy that signal is. In both cases, the policymaker can mitigate the conflict by paying positive transfers for moderate policy and taking them away as policy becomes more extreme.

The key difference between the two models flows from the state-dependence of the conflict of interest. In our model, the policy disagreement between the policymaker and the expert is increasing in the distance between the expert's signal and the (common) prior expectation. (Che and Kartik (2009) make a related point.) As a result, transfers for moderate policy have little direct benefit. But incentive compatibility requires paying transfers at moderate signals if policy at more extreme signals is to be affected. Since there is little policy gain to offset the cost of those transfers at moderate signals, the cost of using transfers to mitigate policy disagreement for more extreme expert information weighs more heavily on the policymaker than it does in the additive-bias model.

1 The Model

There are two actors, a *L*egislator and an *E*xpert. Together, they will determine a policy $p \in [0, 1]$.

Information Structure The optimal policy depends on the state of the world, $\omega \in [0, 1]$. Neither player knows this state. They share a common prior that the state is uniformly distributed on $[0, 1]$.

The Expert, but not the Legislator, observes a signal correlated with the state of the world. This signal is not verifiable, and cannot be shared with the Legislator.

The players agree that, with some probability, the signal is equal to the state, and, with complimentary probability, the signal is the realization of a uniform random variable on $[0, 1]$, independent of the state. The players disagree about the probability that the signal equals the state. Let π^i be player i's probability assessment that the signal equals the state. To capture the Expert's overprecision, assume $\pi^E > \pi^L$.

By Lemma 2 in Appendix A.1, i's posterior expectation of the state is a weighted average of the signal and the prior mean, $1/2$, with weight on the signal equal to i's assessment that the signal equals the state:

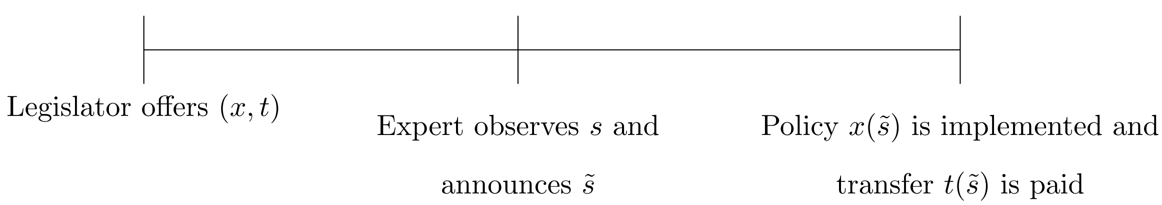
$$\mathbb{E}^i(\omega \mid s) = \pi^i s + (1 - \pi^i)\frac{1}{2}.$$

Contracts We take a mechanism design approach to institutional choice. The policy and a transfer will be determined by the Expert's report about the signal he observes. Denote the reported signal by $\tilde{s}$.

A **mechanism** is a pair (x, t), where $x : [0, 1] \to [0, 1]$ specifies the policy choice as a function of the signal and $t : [0, 1] \to \mathbb{R}_+$ specifies a transfer paid by the Legislator to the Expert as a function of the signal. Notice that t takes nonnegative values; this reflects the limited liability of the Expert.

To justify standard optimal control techniques, we impose some technical conditions on mechanisms. First, x and t are each piecewise continuously differentiable. Second, there is a constant $K > 1$ such that $\left|\frac{dx}{ds}(s)\right| \leq K$ for all s at which t is differentiable. Denote by $\mathcal{M}$ the set of all such mechanisms.

The timing is as follows:



Preferences The Legislator and Expert share the same state-contingent preferences over policies, desiring to maximize the expectation of $-(p - \omega)^2$. Write $u^i(p, s)$ for player i's expectation of this quantity, given signal s. By the Law of Total Variance:

$$u^i(p, s) = -(p - \mathbb{E}^i(\omega \mid s))^2 - \text{var}^i(\omega \mid s). \tag{1}$$

Let i**'s ideal policy at** s be $x^i(s) = \text{argmax}_p\, u^i(p, s)$. Direct calculation gives $x^i(s) = \mathbb{E}^i(\omega \mid s)$.

Transfers enter each player's payoffs quasi-linearly. If the signal is s, the policy is p, and the transfer is t, then the Legislator's payoff is $u^L(p, s) - t$ and the Expert's payoff is $u^E(p, s) + t$.

Interpretation Taken literally, a mechanism is a very particular kind of institution. The Expert simply reports (truthfully or not) the signal, and the policy and transfer are mechanically implemented. But the formalism is actually far more flexible. The famous *Revelation Principle* says that the outcome of any equilibrium in any institution can be replicated by a mechanism in which players maximize utility by reporting the truth.[4] (For a textbook treatment by one of the principles's discoverers, see Myerson (1997, ch. 6).)

The Revelation Principle licenses the following procedure. First, we will characterize the mechanism that maximizes the Legislator's expected payoff, subject to the constraint that the Expert always wants to truthfully reveal her signal. Second, we will present a simple institution that attains the same expected payoff for the Legislator. By the Revelation Principle, we can conclude that that institution is in fact optimal for the Legislator.

2 The Optimal Mechanism

The mechanism (x, t) is **incentive compatible** if:

$$u^E(x(s), s) + t(s) \geq u^E(x(\tilde{s}), s) + t(\tilde{s}) \quad \text{for all } s, \tilde{s}. \tag{2}$$

In words, the Expert prefers to report the true signal s rather than any other signal $\tilde{s}$, no matter what the true s is.

To get a sense of what incentive compatibility implies, it will be useful to look at some examples.

Example 1 Consider an incentive compatible mechanism (x, t) where transfers are identically zero, $t(s) = 0$ for all s. If s is an interior point where x is differentiable, then Inequality 2 implies:

$$\frac{\partial}{\partial p} u^E(x(s), s) \frac{d}{ds} x(s) = 0.$$

Two focal mechanisms satisfy this. In one, the Expert gets the policy $x^E(s)$ (and $\partial u^E / \partial p = 0$). In the other, the policy is constant (and $dx/ds = 0$).

A **delegation window** chooses the expert's optimal policy when the signal lies within an interval $[\underline{s}, \overline{s}]$ and chooses the policy equal to the closest interval endpoint when the signal lies

[4] Stochastic mechanisms are needed to account for mixed strategies. In our model, stochastic mechanisms lead to subtle interactions with the limited liability constraint. We leave exploring these interactions for future work. Thanks to Navin Kartik for helpful discussions on this point.

outside the interval. This mechanism, studied by Holmström (1984) and Epstein and O'Halloran (1994), combines the two possibilities from Example 1.

Example 1's calculation extends to a full characterization of incentive compatibility.

Lemma 1. *The mechanism (x, t) is incentive compatible if and only if:*

1. *x is (weakly) increasing, and*
2. *$t'(s) = -\frac{\partial u^E}{\partial p}(x(s), s) \cdot x'(s)$ at all s where x is differentiable.*

Proof. See Theorems 7.1 and 7.3 in Fudenberg and Tirole (1991). ■

This result opens up possibilities rather different from the standard delegation window.

Example 2 The Legislator's privately optimal policy has $\frac{dx^L}{ds}(s) = \pi^L > 0$. Let:

$$\begin{aligned} t^L(s) &= -\int_0^s \frac{\partial u^E}{\partial p}(x^L(\tilde{s}), \tilde{s}) \cdot \pi^L \, d\tilde{s} \\ &= \int_0^s 2\pi^L(\pi^E - \pi^L)(1/2 - \tilde{s}) \, d\tilde{s} \\ &= \pi^L(\pi^E - \pi^L)(s - s^2). \end{aligned}$$

The mechanism (x^L, t^L) is both incentive compatible (by Lemma 1) and in $\mathcal{M}$.

The mechanism in Example 2 guarantees the Legislator the same policy she would get if she directly observed the state of the world. No simple delegation window can do that. Given that possibility, it comes as something of a surprise that the optimum does not use transfers at all.

The optimal mechanism is determined by the Program:

$$\begin{aligned} \max_{(x,t,v)} \int_0^1 & u^L(x(s), s) - t(s) \, ds \\ \text{st } & x(\cdot) \text{ is weakly increasing} \\ & t'(s) = -\frac{\partial u^E}{\partial p}(x(s), s) \cdot v(s) \\ & x'(s) = v(s) \\ & t(s) \geq 0 \\ & |v(s)| \leq K. \end{aligned} \tag{3}$$

The new control variable v facilitates writing the optimization problem in the canonical form for optimal control; the constraints on that variable and on t ensure that the mechanism is in $\mathcal{M}$.

Proposition 1. *Let* $\underline{s} = \frac{\pi^E - \pi^L}{2\pi^E - \pi^L}$*. The unique solution to Program 3 is* (x^*, t^*)*, where:*

$$x^*(s) = \begin{cases} x^E(\underline{s}) & \textit{if } s \leq \underline{s} \\ x^E(s) & \textit{if } \underline{s} < s < \overline{s} \\ x^E(1-\underline{s}) & \textit{if } 1-\underline{s} \leq s \end{cases},$$

and $t^*(s) = 0$ *for all* s*.*

Proof. See Appendix A.2. ∎

Corollary 1. *A delegation window is an optimal institution.*

Proof. The payoffs from the mechanism in Proposition 1 are attained when the Expert is allowed to choose any policy in the interval $[x^E(\underline{s}), x^E(1-\underline{s})]$. ∎

Figure 1 illustrates the optimal mechanism.

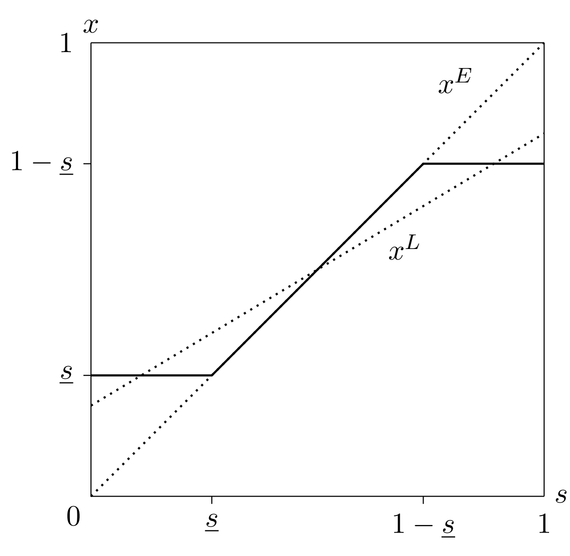


Figure 1. The solid curve is the policy function of an optimal mechanism. The dashed lines are ideal policies.

3 Discussion

When the Legislator believes the Expert suffers from overprecision, monetary transfers are not used in the optimal delegation mechanism. Why not?

A useful first step in thinking about an answer is to consider how transfers could be used. Lemma 1's condition on $t'(s)$ has an important implication. Suppose the mechanism (x, t) has policy more moderate than the Expert's ideal policy: $s < \frac{1}{2}$ implies $x(s) > x^E(s)$ and $s > \frac{1}{2}$ implies $x(s) < x^E(s)$. Since:

$$t'(s) = 2\left(x(s) - x^E(s)\right) x'(s), \tag{4}$$

the transfer to the Expert must be increasing for $s < 1/2$ and decreasing for $s > 1/2$. Intuitively, the Expert's incentives to accept moderate policy arise from positive transfers that are used to boost her payoff at $s = 1/2$ and that are taken away as the reported signal becomes more extreme. Since the policy at s affects the rate of change of the transfer at s, there is a linkage between the policy choice at s and the transfers paid at other signals.

This observation sets up a heuristic argument that no perturbation away from the Expert's ideal policy can increase the Legislator's expected payoff, once the costs of the required transfers are accounted for. We will present the argument for a prior density f that is symmetric about 1/2 and single-peaked, but otherwise unrestricted.[5] This will show that the uniform distribution, while useful for technical reasons, is not driving our results.[6]

Consider a mechanism that implements a delegation window. Fix a signal $\hat{s} \in (\underline{s}, 1/2)$ and small numbers $\epsilon > 0$ and $\delta > 0$. Consider the perturbed mechanism that implements $x^E(s) + \epsilon$ on the interval $(\hat{s} - \delta/2, \hat{s} + \delta/2)$, implements $x^E(s) - \epsilon$ on the interval $(1 - \hat{s} - \delta/2, 1 - \hat{s} + \delta/2)$ and follows the original delegation window otherwise otherwise.[7] We will compute the costs and benefits of the perturbation over the interval $[0, 1/2]$; symmetry implies that the total costs and benefits are twice those calculated below.

[5]Notice that this assumption does rule out the case of very little probability mass near the point with no conflict of interest.

[6]The uniform distribution is used in Lemma 4 in Appendix A.2.

[7]This perturbation does not preserve continuity and monotonicity of the mechanism. It is modeled on the perturbations used in the proofs of the theorems we appeal to in the appendix.

The perturbation improves the Policymaker's expected policy payoff by approximately:

$$\underbrace{\epsilon\frac{\partial u^L}{\partial p}(x^E(\hat{s}),\hat{s})\cdot}_{\text{improved payoff}} \underbrace{f(\hat{s})\delta}_{\text{probability}} = \epsilon\delta\cdot 2(\pi^E-\pi^L)\left(\frac{1}{2}-\hat{s}\right)f(\hat{s}). \tag{5}$$

To interpret this expression, notice that the difference between the Policymaker's and the Expert's ideal policies at $\hat{s}$ is $x^L(\hat{s})-x^E(\hat{s})=(\pi^E-\pi^L)(\hat{s}-1/2)$. Thus the gain is proportional to the difference between the ideal policies times the density of signals at $\hat{s}$.

The perturbation also implies higher transfers. Over the interval $(\hat{s}-\delta/2,\hat{s}+\delta/2)$, Lemma 1 implies that transfers increase linearly from 0 to

$$t(\hat{s}+\delta/2)=\int_{\hat{s}-\delta/2}^{\hat{s}+\delta/2}2(x^E(s)+\epsilon-x^E(s))\pi^E\,ds=\epsilon\delta\cdot 2\pi^E.$$

Because the mechanism implements the Expert's ideal policy on $[\hat{s}+\delta,1/2]$, Equation 4 implies that $t'(s)=0$ on that interval. Thus the additional transfer must be paid to all types between $\hat{s}+\delta/2$ and $1/2$. The overall expected additional transfer paid to types less than $1/2$ is approximately

$$\underbrace{\epsilon\delta\cdot 2\pi^E}_{\text{extra transfer}}\cdot\underbrace{\Pr(s\in[\hat{s},1/2])}_{\text{probability}}=\epsilon\delta\cdot 2\pi^E\left(\frac{1}{2}-F(\hat{s})\right). \tag{6}$$

Combine the approximations in Equations 5 and 6 to approximate the overall change in the Policymaker's expected payoff as:

$$\epsilon\delta\cdot\Delta(\hat{s})\equiv\epsilon\delta\left((\pi^E-\pi^L)\left(\frac{1}{2}-\hat{s}\right)f(\hat{s})-2\pi^E\left(\frac{1}{2}-F(\hat{s})\right)\right)$$

Appendix A.3 shows that $\Delta(s)<0$ for all $s\in[0,1/2)$. This inequality stands on two facts about the payoff consequences of the perturbation. First, for signals close to $1/2$, the conflict of interest between the Policymaker and the Expert is too small to make using transfers worthwhile. Second, for signals far from $1/2$, moderating the implemented policy requires transfers be paid for a large measure of signals.

We can compare this to what obtains in the more usual additive-bias model a la Crawford and Sobel (1982). Krishna and Morgan (2008) solve for the optimal mechanism with transfers in this case. The key difference between the models is that their Legislator and Expert have ideal policies that are parallel shifts of one another. This implies that the Legislator's marginal benefit of moving policy away from the Expert's ideal policy never vanishes, so the analogue to the function Δ in their model eventually turns positive. Substantively, the key difference is that, in our model, the conflict of interest between the Legislator and Expert depends on the signal, and actually vanishes at the neutral news signal $s = \frac{1}{2}$.

This difference highlights that models with different preferences and with different beliefs about the degree of expertise do not capture the same substantive forces. As such, our result should not be taken as a justification for the usual models of delegation in political science. Rather, the result, along with those of Hirsch (2016), suggests that there is much interesting work to be done on the institutional politics of belief-based differences over policy.

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A Online Appendix

A.1 Preliminary Results

Lemma 2. *Fix a signal realization s.*

1. *Player i's posterior distribution on ω is:*

$$F^i(\omega \mid s) = \begin{cases} 0 & \text{if } \omega < 0 \\ (1-\pi^i)\omega & \text{if } 0 \le \omega < s \\ \pi^i + (1-\pi^i)\omega & \text{if } s \le \omega \le 1 \\ 1 & \text{if } 1 < \omega \end{cases} . \tag{7}$$

2. *Player i's posterior expectation of ω is:*

$$\mathbb{E}^i(\omega \mid s) = \pi^i s + (1-\pi^i)\frac{1}{2}.$$

Proof. Part 1 is the example in Section 4 of Macci (1996), specialized to the uniform distribution on $[0,1]$. Part 2 follows by integration. ■

Lemma 3. *u^i is concave in p.*

Proof. Immediate from Equation 1. ■

A.2 Proof of Proposition 1

Recall the optimal mechanism is determined by Program 3. Consider the *relaxed program:*

$$\begin{aligned} \max_{(x,t,v)} \int_0^1 & u^L(x(s),s) - t(s)\, ds \\ \text{st } t'(s) &= -\frac{\partial u^E}{\partial p}(x(s),s)\cdot v(s) \\ x'(s) &= v(s) \\ t(s) &\ge 0 \\ \left|\frac{dt}{ds}(s)\right| &\le K. \end{aligned} \tag{8}$$

We will show that the (essentially) unique solution to the relaxed program is (x^*, t^*, v^*), where:

$$v^*(s) = \begin{cases} \pi^E & \text{if } s \in [\underline{s}, 1-\underline{s}] \\ 0 & \text{otherwise.} \end{cases}$$

Since x^* is weakly increasing, that will imply that (x^*, t^*) is in fact the optimal mechanism.

Writing λ_1 for the co-state on x and λ_2 for the co-state on t, the Hamiltonian is:

$$\mathcal{H}(s, v, x, t, \lambda_1, \lambda_2) = u^L(x, s) - t + \lambda_1 v - \lambda_2 \frac{\partial u^E}{\partial p}(x, s) \cdot v.$$

Writing $\mu \geq 0$ for the multiplier on the state-variable inequality constraint, the Lagrangian is:

$$\mathcal{L}(s, v, x, t, \lambda_1, \lambda_2, \mu) = \mathcal{H}(s, v, x, t, \lambda_1, \lambda_2) + \mu \cdot t(s). \tag{9}$$

The **Pontryagin conditions** are:

$$\frac{\partial \lambda_1}{\partial s} = -\frac{\partial \mathcal{L}}{\partial x} \qquad \text{(Co-state 1)}$$

$$\frac{\partial \lambda_2}{\partial s} = -\frac{\partial \mathcal{L}}{\partial t} \qquad \text{Co-state 2)}$$

$$v \in \arg\max_{v \in [-K, K]} \mathcal{L} \qquad \text{(Static optimality)}$$

$$\lambda_1(0) = \lambda_1(1) = 0 \qquad \text{(Transversality condition 1)}$$

$$\lambda_2(0) = \lambda_2(1) = 0 \qquad \text{(Transversality condition 2)}$$

$$\mu \cdot t(s) = 0 \qquad \text{(Complementary slackness)}$$

Lemma 4. *Fix $(x, t) \in \mathcal{M}$, and let $v(s) = x'(s)$ for all differentiability points of x. Suppose there is a $(\lambda_1, \lambda_2, \mu)$ such that:*

1. *λ_1 and λ_2 are continuous and piece-wise differentiable,*

2. *μ is piece-wise continuous, and*

3. *$(v, x, t, \lambda_1, \lambda_2, \mu)$ satisfy the Pontryagin conditions.*

Then (v, x, t) solves the relaxed program, and any solution to the relaxed program has state trajectories (x, t).

Proof. We apply Theorem 8.2 and Corollary 8.1 in Hartl, Sethi and Vickson (1995). There is only one hypothesis that is not trivial to verify.

The maximized Hamiltonian is:

$$\mathcal{H}^*(s, x, t, \lambda_1, \lambda_2) = \max_{v \in [-K, K]} \mathcal{H}(s, v, x, t, \lambda_1, \lambda_2).$$

Because the Hamiltonian is linear in v, the maximum is either $\mathcal{H}(s, -K, x, t, \lambda_1, \lambda_2)$ or $\mathcal{H}(s, K, x, t, \lambda_1, \lambda_2)$. In either case, Lemma 3 implies that $\mathcal{H}^*(s, x, t, \lambda_1, \lambda_2)$ is concave in x and t. ■

Substituting our functional forms, the first three Pontryagin conditions become:

$$\begin{aligned}
\lambda_1' &= 2(x - \pi^L s - (1 - \pi^L)\frac{1}{2}) - 2\lambda_2 v \\
\lambda_2' &= 1 - \mu \\
0 &= \lambda_1 + 2\lambda_2(x - \pi^E s - (1 - \pi^E)\frac{1}{2})
\end{aligned}$$

Recalling that $\underline{s} = \frac{\pi^E - \pi^L}{2\pi^E - \pi^L}$, it is straightforward to verify the Pontryagin conditions for

$(v^*, x^*, t^*, \lambda_1^*, \lambda_2^*, \mu^*)$, where:

$$\lambda_1(s) = \begin{cases} 2\left(\pi^E(1-\underline{s}) + (\pi^L - \pi^E)\frac{1}{2}\right)(1-s) - \pi^L(1-s^2) & \text{if } s > 1-\underline{s} \\ 0 & \text{if } 1-\underline{s} \geq s \geq \underline{s} \\ 2\left(\pi^E\underline{s} + (\pi^L - \pi^E)\frac{1}{2}\right)s - \pi^L s^2 & \text{if } \underline{s} > s \end{cases}$$

$$\lambda_2(s) = \begin{cases} \frac{\pi^L}{2\pi^E}(1-s) & \text{if } s > 1-\underline{s} \\ \frac{(\pi^E - \pi^L)(s-\frac{1}{2})}{\pi^E} & \text{if } 1-\underline{s} \geq s \geq \underline{s} \\ -\frac{\pi^L}{2\pi^E}s & \text{if } \underline{s} > s \end{cases}$$

$$\mu(s) = \begin{cases} 1 - \frac{\pi^L}{2\pi^E} & \text{if } s > 1-\underline{s} \\ \frac{\pi^L}{\pi^E} & \text{if } 1-\underline{s} \geq s \geq \underline{s} \\ 1 + \frac{\pi^L}{2\pi^E} & \text{if } \underline{s} > s \end{cases} .$$

A.3 Formalizing Section 3

Section 3 gives a heuristic argument for why transfers are not used in the optimal mechanism. This appendix offers a more formal argument, rooted in the necessary conditions for an optimum with positive transfers.

As in Section 3, we will allow for any distribution of signals that has a density, is symmetric about 1/2, and is single-peaked. Write F for the distribution of signals, and f for its density.

Consider a solution to Program 3 in which x is symmetric about $\frac{1}{2}$ and has positive transfers on some interval around $\frac{1}{2}$.

The Pontryagin conditions are:

$$\lambda_1' = 2(x - \pi^L s - (1 - \pi^L)\frac{1}{2})f(s) - 2\lambda_2 v$$

$$\lambda_2' = f(s) - \mu$$

$$0 = \lambda_1 + 2\lambda_2(x - \pi^E s - (1 - \pi^E)\frac{1}{2})$$

$$\lambda_1(0) = \lambda_1(1) = 0$$

$$\lambda_2(0) = \lambda_2(1) = 0$$

$$\mu(s)t(s) = 0$$

Conjecture a solution in which x is symmetric about $\frac{1}{2}$ and, on some interval around $\frac{1}{2}$, transfers are positive. In such a case, $\mu = 0$. Using the second co-state equation, differentiate the static maximization condition to get:

$$0 = \lambda_1' + 2f(s)(x - \pi^E s - (1 - \pi^E)\frac{1}{2}) + 2\lambda_2(x' - \pi^E).$$

Substitute from the first co-state equation to get

$$0 = 2(x - \pi^L s - (1 - \pi^L)\frac{1}{2})f(s) - 2\lambda_2 v + 2f(s)(x - \pi^E s - (1 - \pi^E)\frac{1}{2}) + 2\lambda_2(x' - \pi^E).$$

Since $x' = v$, this simplifies to

$$0 = \left(2(x - \pi^L s - (1 - \pi^L)\frac{1}{2}) + 2(x - \pi^E s - (1 - \pi^E)\frac{1}{2})\right) f(s) - 2\lambda_2 \pi^E.$$

Integrate the second co-state equation to get $\lambda_2(s) = F(s) - K$. Symmetry of x about $\frac{1}{2}$ then implies:

$$0 = \left(2(x - \pi^L s - (1 - \pi^L)\frac{1}{2}) + 2(x - \pi^E s - (1 - \pi^E)\frac{1}{2})\right) f(s) - 2(F(s) - 1/2)\pi^E. \tag{10}$$

Notice that this condition is closely related to the heuristic approach in the main text.

Substituting $x = x^E$ into Equation 10 yields

$$0 = 2(x - \pi^L s - (1-\pi^L)\frac{1}{2})f(s) - 2(F(s) - 1/2)\pi^E$$
$$= -\Delta(s),$$

where Δ is the function defined in Section 3.

Lemma 5. *1. Δ is strictly increasing on $[0, 1/2]$.*

2. $\Delta(1/2) = 0$.

Proof.

1. Differentiate and simplify to get

$$\Delta'(s) = \left((\pi^E - \pi^L)\left(\frac{1}{2} - \hat{s}\right) f'(\hat{s}) + (\pi^E + \pi^L) f(\hat{s})\right) > 0,$$

   where the inequality follows from single-peakedness of f.

2. There are two terms. The first is clearly 0 at $s = 1/2$; symmetry of f implies the second term is also 0 at $s = 1/2$.

■

Corollary 2. *For all $s \in [0, 1/2)$, we have $\Delta(s) < 0$.*

This Corollary has two implications. First, it establishes that Equation 10 does not hold at the Expert's ideal policy. Second, it justifies the claim in Section 3.